- Loss of performances in Figures 6, 8 (Reviewer #1). After investigation, those losses are due to hyperparameter
- choices for the non-convex WDA and OTDA problems. When appropriately selected for each model (decimation
- factor), we obtain running time gain of same orders without compromising performances. In the final version, we will
- add to the supplementary new figures related to the regularization path computations and resulting accuracies.
- Comparison with other solvers (Reviewers #1 and #3). We have considered experiments with Greenkhorn algo-
- rithm but the implementation in POT library and our custom Python version of Matlab Altschuler's Greenkhorn code
- were not competitive with Sinkhorn. Hence, for both versions, Screenkhorn is more competitive than Greenkhorn. The
- computation time gain reaches an order of 30 when comparing our method with Greenkhorn while Screenkhorn is
- almost 2 times faster than Sinkhorn, We will provide this comparison and discussion in the final version.
- On the use of constrained L-BFGS (Reviewers #2 and #3). Our proposed screened dual problem given in (3) or 10 (6) involves explicit box constraints on $e^{u_i^{sc}}$ and $e^{v_i^{sc}}$ (see Proposition 1). Hence, it is a constrained smooth optimization 11
- problem, and standard Sinkhorn's alternating minimization can not be applied. This appears more clearly while writing 12
- its optimality conditions. We resort to L-BFGS-B to solve our constrained convex optimization problem, but any 13
- efficient solver (e.g. proximal based method or Newton method) can be used. Notice that as for the Sinkhorn algorithm, 14
- our Screenkhorn can be accelerated using a GPU implementation of L-BFGS-B [2]. 15
- **Main concerns of Reviewer #2.** Concern 1. The bound in Proposition 3 is similar, up to the additive term ω_{κ} (a 16 discussion about ω_{κ} is provided in below), to the ones found in the literature; in particular for the Sinkhorn algorithm 17
- (see Lemma 2 in [1]) and for the Greenkhorn algorithm (see Corollary 3.3 in [4]). More formally, letting $\{(u^k, v^k)\}_{k\geq 1}$ 18
- denote the iterates returned by the Sinkhorn or the Greenkhorn algorithm, they have $\Psi(u^k, v^k) \Psi(u^\star, v^\star) = \mathcal{O}(RE^{\bar{k}})$ 19
- 20
- where $E^k = ||B(u^k, v^k)\mathbf{1} \mu||_1 + ||B(u^k, v^k)^{\top}\mathbf{1} \nu||_1$, and $R = C_{max}/\eta + \log(n) 2\log(c_{\mu\nu})$ which comes from an upper bound for the ℓ_{∞} -norm of the optimal pair solution (u^*, v^*) of Sinkhorn divergence. In our case, supposing 21
- that n=m and acknowledging that $\log(1/K_{min}^2)=2C_{max}/\eta$, we have $R=C_{max}/\eta-3.5\log(c_{\mu\nu})$. Additionally, 22
- we give in Proposition 2 a bound on E^k that becomes small as the sample budget increases.
- Concern 2. The new formulation (3) has the form of $(\kappa \mu, \nu/\kappa)$ -scaling problem under constraints on the variables
- u and v and the problem is not invariant anymore. This differs significantly from the standard scaling-problems [3], 25
- though the sought transportation map P takes a matrix-scaling form. We further emphasize that κ plays a key role (that 26
- we will emphasize in the final version) in our screening strategy for the dual of Sinkhorn divergence. Indeed, without 27 κ , e^u and e^v can have inversely related scale that may lead in, for instance e^u being too large and e^v being too small, 28
- situation in which the screening test would apply only to coefficients of e^u or e^v and not for both of them. In addition 29
- note that given n, the bounds in Propositions 2 and 3 are derived using the following control of the parameter ε , which
- - is induced by the screening test's construction (4), $c_{\mu\nu}^{1/4}/\sqrt{n} \le \varepsilon \le 1/\sqrt{nK_{\min}}$.
- Concern 3. An explicit form of ω_{κ} (with $\omega_1=0$) is given in L449 of the paper. In the setting of n=m and using 32
- the upper bounds of $||u^{sc}||_{\infty}$ and $||v^{sc}||_{\infty}$ in L447, we derive the following bound: $\omega_k \lesssim R'((1-\kappa)||\mu^{sc}||_1 + (1-\kappa)||\mu^{sc}||_1 + (1-\kappa)||\mu^{sc}||_1$ 33
- κ^{-1}) $||\nu^{sc}||_1$) where $R' = C_{\text{max}}/\eta 0.5\log(n) 0.5\log(c_{\mu\nu})$. A control for the ℓ_1 -norms of the screened marginals 34
- μ^{sc} and ν^{sc} are given in Equations (18) and (19) in Lemma 3. Using the bound of the term ω_{κ} , we will clarify the 35
- bound in Proposition 3 for the final version of the paper.
- Minor comments (all Reviewers). The final version of the paper will include all suggested modifications. 37

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