

1 We thank the reviewers for their comments. We will incorporate all points and suggested clarifications. We assume a  
2 ground set of size  $n$ . Note that set functions are inherently  $2^n$ -dimensional. The goal of the paper is to provide a novel,  
3 mathematically sound, CNN architecture with prototypical evaluation that the community can build on. We first answer  
4 a common question about the complexity; we will include the detailed derivation in the final version.

5 **Complexity analysis** We consider a powerset convolutional layer with  $n_c$  input channels and  $n_f$  output channels.  
6 Convolution is done efficiently in the Fourier domain, i.e.,  $h * s = F^{-1}(\text{diag}(\bar{F}h)Fs)$ , which requires  $\frac{3}{2}n2^n + 2^n$   
7 operations and  $2^n$  floats of memory.

8 *a. Operations:* forward pass:  $n_c n_f (\frac{3}{2}n2^n + 2^n)$  operations, backward pass:  $2n_c n_f (n2^n + 2^{n+1})$  operations.

9 *b. Memory:* forward pass:  $n_c 2^n + n_f 2^n + \#(\text{params.})$  floats, backward pass:  $n_f 2^n + n_c 2^n + \#(\text{params.})$  floats.

10 *c. Parameters:* Using  $k$ -hop filters, a layer requires  $n_f$  bias terms and  $n_c n_f \sum_{i=0}^k \binom{n}{i}$  filtering coefficients.

11 *d. Baseline:* Graph convolutional layers for a hypercube are a special case of powerset convolutional layers (1.250-252).  
12 Hence, they are in the same complexity class. A  $k$ -hop graph convolutional layer requires  $n_f + n_c n_f (k + 1)$  parameters.

13 **Improvements** Using modern GPUs, ground sets up to size  $n \equiv 30$  are feasible [A]. Our TensorFlow implementation  
14 is a prototype meant to demonstrate viability, and thus has limited efficiency. Future work could leverage techniques for  
15 NN dimension reduction, e.g. [B], to scale powerset CNNs to larger domains.

#### 16 **Reviewer 1**

17 Q. *What is the norm on  $s : 2^N \rightarrow \mathbb{R}$  (used e.g. 1.99-100)?*

18 R.  $\|s\| = (\sum_{A \subseteq N} s_A^2)^{1/2}$ .

19 Q. *What is the difference between the two proposed models (1.232)?*

20 R.  $*$ -PCNs are shift-equivariant w.r.t.  $s \mapsto (s_{A \setminus Q})_{A \subseteq N}$  and  $\diamond$ -PCNs w.r.t. its dual shift, i.e.,  $s \mapsto (s_{A \cup Q})_{A \subseteq N}$ .

21 Q. *It would be nice to see a benchmark with more than one-hop filters if doable.*

22 R. As we are filtering in Fourier domain even  $n$ -hops are doable. We ran the benchmark using 2-hop filters and saw  
23 only a small improvement only in some cases. This is likely due to the small scale of our prototypical experiments.

24 Q. *How would the proposed method specialize to graphs and how would it compare to classical GNNs?*

25 R. A weighted graph is a special set function with values only on the two element sets (the edges). Using, e.g.,  
26  $(h * s)_A = \sum_{Q \subseteq N} h_Q s_{A \setminus Q}$ , the powerset CNN would create nonzero values for larger sets (of nodes), i.e., turning it  
27 into an edge-weighted hypergraph, increasing the dimension of the data, in contrast to graph NNs.

#### 28 **Reviewer 2**

29 Q. *Real applications, e.g., in the scope of sensor- or ad-placement, would significantly strengthen this work.*

30 R. These two tasks are subset-selection tasks, in which a set function serves as a tool to assess the quality of subsets,  
31 e.g., by assigning a score to each subset. As a consequence, the set function problems considered in these area are 1.  
32 finding the subset with the highest score subject to some constraints [17, 24] and 2. learning the scoring function [45].  
33 Problem 1 is not a learning problem and Problem 2 is a transductive learning task. Therefore, the proposed method  
34 does not directly apply, and instead would require to be specialized to the transductive setting (if possible).

#### 35 **Reviewer 3**

36 Q. *What are the novel contributions of this paper, and what is prior work [31)?*

37 R. Convolution and associated Fourier transforms were defined in [31] as cited. However, we are the first to extend  
38 these results to design and apply powerset CNNs. This includes the definition of convolutional and pooling layers, the  
39 analysis of patterns matched, and a prototypical implementation and evaluation to show viability. The contribution is  
40 somewhat similar to graph CNNs (e.g. [9]), which built on long existing results from algebraic graph theory.

41 Q. *Showing that powerset CNNs can solve tasks defined on set functions better than the baselines, and that they are  
42 indeed superior to graph-convolutions for those tasks.*

43 R. To our best knowledge there is no prior work on set function classification. Our baseline—viewing them as data  
44 indexed by an undirected hypercube graph—is thus also novel. These graph convolutions are a small subset of the  
45 powerset convolutions based on the symmetric shift in Equation (6), for which we did not include experiments. We  
46 showed, prototypically, that the directed shifts (adding or subtracting an element) can yield improvements and are thus  
47 viable for applications.

48 Q. *Could the proposed powerset CNNs be applied to convolution-deconvolution networks that would allow set function  
49 learning and transformation?*

50 R. (Not in the paper) We successfully trained fully convolutional 1-hop and  $n$ -hop powerset CNNs to solve a similar  
51 task, namely, to transform probability mass functions  $p$ , with  $p_A = p_x$  for  $A = \{x\}$  and  $p_A = 0$  otherwise, to their  
52 associated probability measures  $P_A = \sum_{x \in A} p_x$ . We claim that it is possible to utilize our layers within a variational  
53 autoencoder and, thus, to learn to sample set functions from a target distribution. The challenges in doing so are 1. to  
54 define a bottleneck, e.g., through pooling, 2. a corresponding scheme to undo the dimensionality reduction, and 3. to  
55 find training data. These autoencoders could find application in simulation frameworks used in combinatorial auctions  
56 [13] and to generate submodular functions. We are not sure whether such an architecture would be suitable for Boolean  
57 function synthesis or transformation, as it would require truth-tables as inputs/outputs rather than Boolean expressions.

58 **References** [A] Yi Lu: “Practical tera-scale Walsh-Hadamard Transform.” In FTC 2016. [B] Hackel et al.: “Inference,  
59 Learning and Attention Mechanisms that Exploit and Preserve Sparsity in CNNs.” In GCPR 2018.