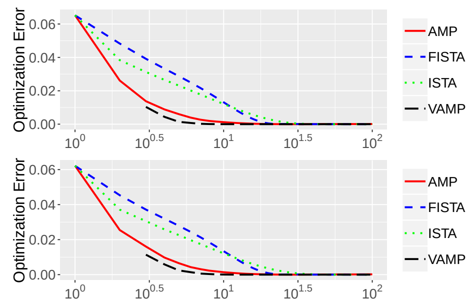
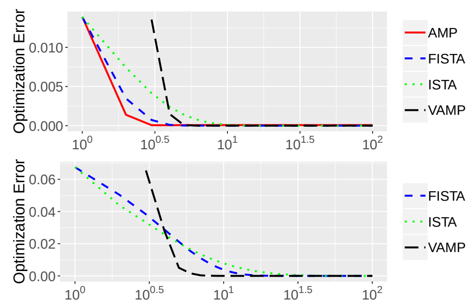
Thanks to the reviewers for the insightful and constructive feedback. It will surely improve the manuscript. Due to space constraints, instead of responding point-by-point, we address points in common with multiple reviews. All minor comments made by reviewer #1 have been addressed and incorporated into a revised version of the paper.

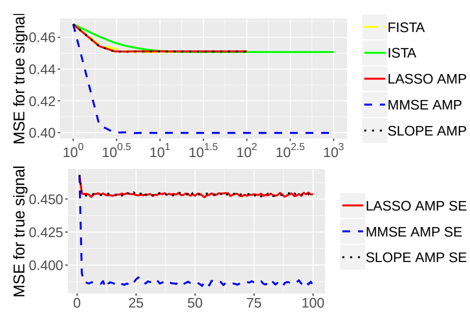
i.i.d. Gaussian measurement matrix assumption. While, in general, AMP theory provides performance guarantees only for i.i.d. sub-Gaussian data, in practice, favorable performance of AMP seems to be more universal. For example, in Fig. 1a, we illustrate the performance of AMP for i.i.d. zero mean, $1/n$ variance design matrices that are *not* Gaussian (one i.i.d. ± 1 Bernoulli (top) and one i.i.d. shifted exponential (bottom)). In both cases, AMP converges very fast, thus demonstrating its robustness to distributional assumptions.



(a) i.i.d. ± 1 Bernoulli design matrix (top) and i.i.d. shifted exponential design matrix (bottom)



(b) i.i.d. Gaussian design matrix (top) and non-i.i.d. right rotationally-invariant design matrix where AMP diverges (bottom)



(c) i.i.d. Gaussian design matrix

Figure 1: Performance of AMP variants in different settings with Bernoulli-Gaussian prior, dimension = 1000, and sample size = 300.

Recent work proposes a variant of AMP, called vector-AMP or VAMP, which is a computationally-efficient algorithm that *provably* works for a wide range of design matrices, namely, those that are right rotationally-invariant. We thank reviewer #2 for pointing us to "AMP for convex optimization with nonseparable penalties" by Manoel et al, which studies VAMP for a similar setting as SLOPE. However, the type of nonseparability considered in the referenced work requires the penalty to be separable on subsets of an affine transformation of its input. As such, the setting does not directly apply to SLOPE, but we have built a hybrid, "SLOPE VAMP", based on code generously shared by the authors of the referenced work, which performs very well in the (non-) i.i.d. (non-) Gaussian regime (see Fig. 1a and 1b). Motivated by these promising empirical results, we feel that theoretically understanding SLOPE dynamics with VAMP is an exciting direction that we plan to pursue in other work.

Known signal prior assumption. We are intrigued by reviewer #3's suggestion of using EM- or SURE-based AMP strategies to remove this assumption. We would like to pursue this within the SLOPE framework, though we haven't done so at this time. We believe that developing such strategies alongside SLOPE VAMP would provide a quite general framework for recovery of the SLOPE estimator.

Comparison to MMSE AMP. In general, the (statistical) motivation for using methods like LASSO or SLOPE is to perform variable selection, and in addition, for SLOPE, to control the false discovery rate. Both methods are therefore biased and, consequently, MMSE AMP strategies will, by design, outperform *if performance is based on MSE*. To combine the best of both methods, one could also incorporate a "debiasing" device in SLOPE AMP, à la "Debiasing the LASSO: optimal sample size for Gaussian designs" by Javanmard & Montanari, but we will leave this for future work. Nevertheless, Fig. 1c suggests that SLOPE AMP has MSE that is not too much worse than MMSE AMP.

Comparison to [20] and [12]. While [20] have the same asymptotic analysis, we have a clear, rigorous statement of where it applies. That is, the analysis in [20] applies *if* the state evolution has a unique fixed point, and our Thm. 1 states precise conditions under which this is true. Moreover, we believe that our algorithmic approach offers a more concrete connection between the finite-sample behavior of the SLOPE estimator and its asymptotic distribution. We also agree with reviewer #2 that a discussion of the main results of [12] to highlight the gap between optimal estimators and any convex penalty would be useful for readers and we will add it to the final manuscript.

Real-world data. We agree with reviewer #1 that performing an empirical study on real-world data will significantly strengthen our results. Previous research has tested SLOPE performance on Genome-Wide Association Studies (GWAS) data (see "The Northern Finland Birth Cohort of 1966 (NFBC)"). Due to its precedence in the SLOPE literature and its inherent scientific importance, we intend to test SLOPE AMP on this data, however, there are restrictions on its use and we are currently undergoing the protocols needed to be granted access to the data by the NIH.