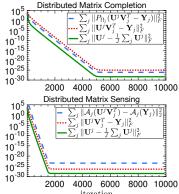
We appreciate the elaborate and constructive responses of the reviewers.

Reviewer 1:

Q1: ...the proposed method on distributed systems is not validated by experiments A1: We initially presented only experiments for distributed matrix factorization (DMF) in the introduction (Fig. 1 and lines 78–82) with details in Section 13 of the supplement. We have now also conducted experiments on distributed matrix completion (DMC) (the same setup as in DMF, except that only $3r \max\{m,n\}$ entries of the rank-r matrix $Y \in \mathbb{R}^{n \times m}$ are observed) using DGD+LOCAL with random initialization. As shown at right, the objective value, recovery error, and consensus error all converge quickly to 0. Similar results are shown for distributed matrix sensing (DMS). We will incorporate these experiments into the final paper.



Q2: Novelty not clear... connection to existing works...

A2: We agree that many of the results in Section 2.2.1 are from existing works as we cited, but we note that these are not our main results. Since the objective function in DMF does not satisfy the common assumption of a globally Lipschitz gradient, Theorem 2.4 extends Theorem 2.3 for functions with a locally Lipschitz gradient.

That being said, we realize that we did not communicate our work with adequate precision. Our first main contribution 16 17 comprises the algorithmic and geometric results for DGD+LOCAL: (i) Section 2.2.2 shows that DGD+LOCAL will converge to a second-order critical point of the regularized objective function (7), and (ii) Section 2.3 provides 18 conditions under which the geometric landscape of the distributed objective function (7) is "equivalent" to the geometric 19 landscape of the original centralized objective function, ensuring exact consensus of DGD+LOCAL, in contrast to 20 general DGD results which admit consensus error proportional to the stepsize [15,23]. Our second main contribution 21 is the result in Section 3 showing consensus and global optimality of DGD+LOCAL for DMF. In the revision, we will 22 highlight the novelty and contributions w.r.t. previous works more clearly in the introduction. 23

Q3: Theorem 2.5,... under the assumption that $\{z(k)\}$ is bounded... do not prove when this condition will be satisfied. A3: By assuming only Lipschitz gradient and the KL inequality, in general one cannot guarantee boundedness. Thus, it is commonly assumed that the sequence is bounded, e.g., in [1, 2] and Theorems 2.1–2.2. If we further assume that the function is coercive, then the generated sequence is bounded. We will incorporate this discussion in the revision.

28 Q4: In the literature of MF, there have been a number of works... such kind of statistical guarantee is missing.

A4: We think the reviewer may be referring to statistical guarantees for matrix completion. This paper focuses on MF, but the experiment in Q1 demonstrates the potential to extend DGD+LOCAL results to DMC. In this case, we believe the existing statistical guarantees for DMC can be directly applied thanks to the equivalence of the geometric landscape between the centralized and distributed objective functions. This is the subject of future work.

Reviewer 2: We appreciate the positive comments and will polish the presentation of the theorems.

Reviewer 3:

34

35

53

Q1: Overall, the paper is well written and easy to follow. However, I would encourage to highlight contributions ...

36 A1: This is a great suggestion and we will highlight our results in a more transparent way, as also requested by R1.

37 Q2: The theorems all look sound; however, only asymptotic results are provided.

A2: The (asymptotic) convergence rate (which is at least sublinear depending on the KL exponent) of DGD+LOCAL can be obtained by using the KL framework in [1,2], as used in [Proposition 2, 24]. Surprisingly, both Fig. 1 and the top right figure suggest that DGD+LOCAL for DMF converges at a *linear* rate. We will incorporate this discussion and leave the investigation to future work.

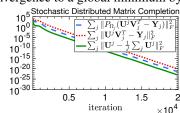
42 Q3: A discussion of such alternative approaches ... and comparison would clearly strengthen the paper...

A3: Compared with primal-dual methods which require a "star topology", DGD+LOCAL (or DGD) can be applied to ring networks without a central node. DGD+LOCAL is similar in spirit to gossip-based methods, but differs in the order of performing the local update and local averaging steps. This small difference allows us to view the proposed algorithm as performing GD on a regularized objective function and thus prove convergence to a global minimum by

47 geometric analysis of that function. We will incorporate this discussion.

this and similar discussion for DMS.

48 Q4: allowing for stochastic gradient updates could increase the impact
49 A4: This is a great suggestion. Note that the geometric analysis in Section 2.3 is for
50 the objective function and can be utilized to guarantee convergence of any iterative
51 algorithm. We apply "Stochastic DGD+LOCAL" (in each iteration we randomly
52 chose one node to update) to DMC and show the result at right. We will incorporate



Q5: Could the authors comment to connections to the work [Matrix Completion has no Spurious Local Minima]?

A5: We apologize for the omission. Similar to [8,18], that work provides geometric analysis of the centralized MC problem, while part of our work focuses on the geometric analysis of the distributed problem. By connecting these two formulations, we show the distributed problem must inherit the same benign geometry.