

1 We thank the reviewers for the feedback. We will ensure that all the minor issues are corrected in the final version of the  
2 draft. Please find the responses for other comments/queries below.

3 R2 • Explain the identity  $\inf_{c \in [0,1]} \frac{1}{2}(c + \frac{a}{c}) \leq \sqrt{a} + a/2$ .  
4 – if  $a \leq 1$ , take  $c = \sqrt{a}$  and the inf is less than  $\sqrt{a}$  and thus less than  $\sqrt{a} + a/2$ ;  
5 – if  $a \geq 1$ , take  $c = 1$  and the inf is less than  $1/2 + a/2$ , which is less than  $\sqrt{a} + a/2$  because  $1/2 < \sqrt{a}$   
6 • Please be specific about what part of [31] is being referenced in line 166. It is Theorem-6.7 of page 336 of [31],  
7 for the randomized version. We will be more precise in the final version.

8 R3 • Provide an outline table to clearly illustrate how the overall algorithm looks like: To solve the original problem,  
9 does one have to solve SFMC<sub>i</sub> iteratively? To solve SFMC<sub>i</sub>, does one have need algorithm 1 and solving SFMD<sub>i</sub>  
10 several times? We mentioned the gist of the algorithms in line 66-69.

11 "In Section 3.1, we show that the continuous optimization problem SFMC<sub>i</sub> can be optimized using discrete  
12 oracles SFMD<sub>i</sub> using a modified divide-and-conquer algorithm [15]. In Section 3.2, we use various optimization  
13 algorithms that use SFMC<sub>i</sub> as the inner loop to solve the continuous optimization problem in Eq.(3)  
14 consequently solving the SFM problem in Eq.(1)."

15 We would be glad to reiterate this before section 3, in particular in more "algorithmic" form, if this improves  
16 the clarity of the paper.

17 • The block coordinate descent method (BCD) discussed in Section 3.3 is cyclic BCD. The BCD in section 3.4 is  
18 still cyclic, while it is a reduced version for the case  $r=2$ . The authors would like to check the RCD method and  
19 its accelerated version in references [17,18]. Although each step of descent therein is to compute projection  
20 to base polytope which is different from this work due to the new proximal term on  $w$ , the authors would  
21 like to give a discussion on whether that method may be used here or even provide additional experimental  
22 evaluations on RCD. This may easily be extended to Randomized coordinate descent method [17,18]. However,  
23 the possibility of extending it to the accelerated version of the randomized coordinate descent [17] is non-trivial  
24 and need some more time.

25 • The experiments only consider chain-graphs as decomposed parts. In reference [15, 18], the DSFM problems  
26 may include much more complex potentials defined over superpixels. Please provide the results with such  
27 settings. We ll try to add results for a problem where one of the summand functions as a concave function over  
28 super pixels instead of just chain or frame functions in the final version of the draft.

29 • Other than just the number of iterations, it would be better to have CPU times of different algorithms for  
30 comparison as well.

31 The CPU times are proportional to the number of SFM calls. We would be glad to add the CPU times in the  
32 final version of the draft. The CPU times for 3D SFM decomposed into 3 functions is BCD - 17.40 sec,  $\Delta$   
33 - 15.60 sec,  $\Delta/t$  - 10.09 sec,  $\Delta/\sqrt{t}$  - 12.27 sec respectively. Note that the timings might change with better  
34 hardware with high parallelism. These timings are on a 4-core laptop processor(Intel(R) Core(TM) i7-7600U  
35 CPU @ 2.80GHz).

36 • A recent work "Revisiting Decomposable Submodular Function Minimization with Incidence Relations" by  
37 Li et al. in NeurIPS 2018 also considered using different terms of  $w$  to accelerate the algorithm for DSFM  
38 problems. Thank you for the reference, we will add it. This seems like an interesting direction for future work  
39 of being able to further speed up the process and come up with better algorithms

40 R4 • For the theoretical results, can you get a faster rate than previous algorithms, say, [15] and [17]? Meanwhile, is  
41 it possible to give oracle complexity in terms of either discrete minimization or function value evaluations? This  
42 would be a good way to theoretical justify the advantage of your algorithm. We do not yet have a conclusive  
43 proof that can show faster convergence rate than the previous algorithms [15,17]. However, the number of  
44 discrete minimization oracle calls is  $O(|U|)$ , where  $U = A_- \setminus A_+$  in Algorithm 1 which is empirically much  
45 lesser than  $O(|V|)$  for smaller  $\varepsilon$ , but for which we currently do not have a bound.

46 • The choice of  $\varepsilon$  depends on diameter of the base polytope  $\Delta$ . Is there an efficient method to estimate it? From  
47 the experiments you observe a tradeoff of choosing  $\varepsilon$ , do you have some heuristic method to choose it. We  
48 suggest to use  $\varepsilon$  proportional to  $\frac{\Delta}{\sqrt{nt}^\alpha}$  to take it as small as possible while only losing a factor of 2 in the  
49 convergence bound, where  $\Delta_i^2 = \sum_{j=1}^n [F_i(\{j\}) + F_i(V \setminus \{j\}) - F_i(V)]^2$  thus, in the result above, we have  
50  $\Delta^2 = r \sum_{i=1}^r \Delta_i^2$ , where  $r$  is the number of summand functions and  $\alpha = 2$ . Therefore,  $\Delta$  may be estimated in  
51  $O(n)$  function evaluations.  $\Delta$  may be approximated for cut functions using weights on the edges( $E$ ) of the  
52 graphs using  $O(|E|)$  operations.

53 • Do you see a hope of applying this technique to minimizing submodular functions over continuous domains?  
54 We certainly believe this technique may be used to minimize continuous submodular functions and approximate  
55 submodular functions. This is the future area of research we are looking into.