

1 We really appreciate the reviewers' time and effort. Thank you for your detailed feedback, thoughtful suggestions and  
2 valuable recommendations for improving the paper! The reviewers appreciate our strong empirical results and mainly  
3 asked us to better situate our work. We respond in more detail below, and will take all comments into account in our  
4 revised version. Additionally, we will shortly release open-source PyTorch implementations of the Hyperbolic Graph  
5 Convolution Networks (HGCM) model and baselines, along with our detailed reproducible training setup.

6 **Contributions (R1, R2, R3):** We sincerely thank R1 for their careful read and for pointing out ambiguities in our  
7 paper. First, our paper is an empirical paper. Our goal is to develop practical techniques that can improve the predictive  
8 performance of recently-developed Graph Convolutional Networks (GCNs) using ideas from hyperbolic geometry.  
9 Our main result is that HGCM achieves error reduction of up to 63.1% in ROC AUC for link prediction and of up to  
10 47.5% in F1 score for node classification. Moreover, using standard notions from hyperbolic geometry (Gromov's  
11  $\delta$ -hyperbolicity), we show that performance is indeed improved when the underlying graph is more "hyperbolic-like".

12 Simply setting GCN variables to be optimized in hyperbolic space does not yield good performance. Our work validates  
13 that three algorithmic ideas based on hyperbolic geometry are important to obtaining predictive accuracy and good  
14 runtime performance: trainable curvature, attention-based hyperbolic aggregation, and optimization on the Lorentz  
15 (hyperboloid) manifold. Our aggregation method relies on two crucial techniques which result in improved performance  
16 compared to standard aggregation in the tangent space at the origin: (1) aggregation is performed at the *local* tangent  
17 space of each point, which better approximates the local hyperbolic geometry; (2) attention scores are computed from  
18 the origin, which allows HGCM to capture node hierarchies. Furthermore, trainable curvature intuitively helps find the  
19 right amount of curvature, and potentially alleviates numerical errors that might arise from limited machine precision.  
20 We show in ablation analysis that these algorithmic contributions result in up to 9.9% absolute gain compared to simple  
21 GCN in hyperbolic space, an improvement that is larger than what any Euclidean GCN variant achieves.

22 **Presentation (R1, R2, R3):** The results in Section 3 are not considered part of our contribution and we now realize that  
23 we could make our claims and presentation more crisp. In our updated draft, we will state known facts from hyperbolic  
24 geometry as propositions rather than corollaries, move standard results to the Appendix as suggested by R2, and more  
25 carefully cite the related literature, including [1], as suggested by R1. We have also fixed the typo in Equation 15 in the  
26 Appendix pointed out by R1 and we confirm that we have been using the correct formulation of parallel transport in our  
27 experiments. Indeed, we had run unit tests verifying that points are mapped to the correct tangent space. We also thank  
28 R1 for noticing the notation error regarding the hyperbolic radius  $i\sqrt{K}$ , and we will make this consistent in the revised  
29 version.

30 We thank R2 for the thorough feedback and will clarify our intuitive explanation on the hyperbolic volume growth  
31 property in line 27, which was meant to illustrate the fact that the volume of balls in Euclidean space grows polynomially  
32 with respect to the radius, while in hyperbolic space it grows exponentially. We will also replace the hyperboloid  
33 manifold by the Lorentz manifold in the revised version.

34 R3 also asks about the correctness of Equation 10, which maps tangent spaces with different curvatures. In order  
35 to apply the exponential map at the north pole, we need to make sure that points are located in the corresponding  
36 tangent space. Fortunately, tangent spaces of the north pole are shared across hyperboloid manifolds that have different  
37 curvatures, making equation 10 mathematically correct as long as  $\sigma(0) = 0$ , which is true for the ReLU activation. We  
38 will make this more explicit in the revised version.

39 **Experiments (R1, R2):** R1 rightly points out that the number of hyper-parameters should be the same for fair  
40 comparison between models. In our experiments, we were very careful about this and we ensured fairness of all  
41 methods by controlling the number of hyper-parameters and trainable parameters. We will clarify this detail in the  
42 revision. In particular, curvatures in HGCM are trainable parameters which are therefore not subject to hyper-parameter  
43 search. Regarding bias and temperature hyper-parameters, we consistently used the Fermi-Dirac decoder to predict link  
44 probabilities for our method and all baselines (hyperbolic and Euclidean), observing that it performs the same as the  
45 standard dot product decoder commonly used in Euclidean baselines.

46 R2 requests clarification regarding the stability of the hyperboloid model compared to the Poincaré model. In our  
47 experiments, we used both models observing that they achieve similar performance, but that the hyperboloid model  
48 offers more stable optimization. This statement is also supported by the fact that the Poincaré distance function is  
49 numerically unstable due to the denominator term, confirming a previous observation in [2]. We will clarify this point  
50 in the revision.

51 **Connection to prior work (R2):** We appreciate R2's suggestion to discuss the connections to Hyperbolic Neural  
52 Networks (HNN). The main differences are the use of trainable curvatures, the attention-based hyperbolic aggregation  
53 mechanism, and the use of hyperboloid model. In the revised manuscript, we will further highlight and stress these key  
54 components and add an experiment using HNN in the hyperboloid model, as suggested by R2.

55 [1] M. Law, R. Liao, J. Snell, and R. Zemel. "Lorentzian Distance Learning for Hyperbolic Representations." ICML 2019.

56 [2] M. Nickel, and D. Kiela. "Learning Continuous Hierarchies in the Lorentz Model of Hyperbolic Geometry." ICML 2018.