We would like to thank all three reviewers for their constructive comments and suggestions which will help us improve

the quality of this paper (ID: 1916). 2

Reviewer #1: 3

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Q1: I would expect more explanations of the main result, e.g., a proof sketch with some

intuition about the significance of this result.

A1: Thank you for pointing this out. We will add a new paragraph or subsection after we

present Theorem 2.1 to include a proof sketch with some intuition about the significance

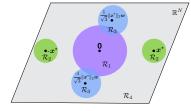
of our result. In this section, we will also include the estimation error bounds obtained Fig.1: estimation error bound.

from Theorem 2.1 and some discussion of the parameters ϵ and η as requested by reviewers #2 and #3.

Q2: In the Applications section, constants can be hidden. I would suggest the 10 authors hide details like the division of regions and formulas in line 196 to make 11 more space for the main result section. 12

A2: We would like to emphasize that the constants and formulas in line 196 are 13 critical. These constants are carefully chosen to argue that the two applications 14 satisfy the requisite assumptions. Similarly, we feel the partition regions are 15 also very important, and we did receive positive comments from reviewers 16 17

#2 and #3 concerning the paper organization. So, we are hesitant to remove these details. However, we will be allowed a ninth content page if our paper is



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Estimation error 2.0 2.0 2.0

Fig.2: partition regions in phase retrieval.

accepted, and we plan to devote this page to extending the discussion in the main result section. If that turns out to be 19 insufficient, we plan to reduce the lemmas in the application section into two informal lemmas, and move the current 20 formal lemmas to the supplementary material. 21

Q3: Do experiments on more general settings for the first application. 22

A3: In the revised version, we will incorporate Fig.1, which shows the distance 23 between the local minima of population and empirical risk in the problem of 24 matrix sensing with k = 2, $r = \bar{3}$, and N = 8. 25

Q4: Have a figure with partition regions for phase retrieval. Use a figure to 26 illustrate the assumptions or results. 27

A4: We will add Figs. 2 and 3 in the revised version. Fig. 3 is used to illustrate 28 the assumptions in the example of phase retrieval. In particular, we set the 29 parameters in Example 1.2 as $N=1, x^*=1$, and M=30. We display the 30 population risk and empirical risk together with their gradients and Hessians in 31 Fig. 3. One can see that in the small gradient region (the three parts between the 32 light blue vertical dashed line), $|\lambda_{\min}(\text{hess } g(\mathbf{U}))|$ (which equals the absolute 33 value of the Hessian since N=1) is bounded away from zero. With enough

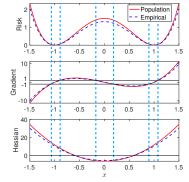


Fig.3: phase retrieval with N=1.

measurements, e.g., M=30, the gradients and Hessians of the empirical and population risk are close to each other. In 35 addition, we think our simulation figures are good illustrations of the main results in Theorem 2.1, i.e., the relationship 36 between critical points of empirical and population risk. We will add more details on this in the main result section. 37

Reviewer #2: 38

We appreciate the positive comments and will add a detailed discussion on how to obtain estimation error bounds.

Reviewer #3:

Q1: The main assumption in the paper does not include degenerate minima, which is surprising (a drawback). 41

A1: A degenerate local minimum of the population risk can correspond to a strict saddle point of the empirical risk due 42 to its randomness. Thus, we do not believe there exists a correspondence in the case of degenerate local minima. 43

Q2: Add an intuitive description of the meaning of ϵ and η in the examples of matrix sensing and phase retrieval and an 44 explanation of the proof strategy for obtaining the parameters. Is there a standard way to compute ϵ and η ? 45

A2: For phase retrieval, note that $|\lambda_{\min}(\nabla^2 g(x))|$ and $||\nabla g(x)||_2$ roughly scale with $||x^*||_2^2$ and $||x^*||_2^3$ in the regions near critical points, which implies that η and ϵ should also scale with $\|x^*\|_2^2$ and $\|x^*\|_2^3$, respectively. For matrix sensing, in a similar way, $|\lambda_{\min}(\text{hess }g(\mathbf{U}))|$ and $\|\text{grad }g(\mathbf{U})\|_F$ roughly scale with λ_k and $\lambda_k^{1.5}$ in the regions near 47 48 critical points, which implies that η and ϵ should also scale with λ_k and $\lambda_k^{1.5}$, respectively. We will incorporate a more detailed discussion on this in the revised version. To obtain these parameters, one can lower bound $|\lambda_{\min}(\text{hess}(g))|$ in a 49 50 small gradient region. In this way, one can adjust the size of the small gradient region to get ϵ , and use the lower bound 51 for $|\lambda_{\min}(\text{hess}(q))|$ as η . In the case when it is not easy to directly bound $|\lambda_{\min}(\text{hess}(q))|$ in a small gradient region, 52 one can also first choose a region for which it is easy to find the lower bound, and then argue that the gradient has a 53 large norm outside of this region, as we did in this paper.