

1 *We thank all the three reviewers for their constructive feedback. Please find our answers to major questions raised.*
2 *Other points will be dealt with in the revised version. Code will be made available by the camera-ready deadline.*

3 **Intuitions and flexibility of the proposed approach (Reviewer #1 and #2):** We agree that we should have provided more
4 intuition for the newly introduced density in the main document. The density builds up on Beta distributions as they
5 are the marginals of the Dirichlet distributed random variable $U \sim \text{Dir}(\alpha)$. We then multiply U with an independent
6 random variable $G \sim \text{Beta}(a, b)$. The resulting random variable $Y = UG$ follows a Beta-Liouville distribution, which
7 allows to account for negative dependence, inherited from the Dirichlet distribution through a Beta stick-breaking
8 construction, as well as positive dependence via a common Beta-factor. Our contribution is now how to transform this
9 distribution that lives within the simplex to one that has support on the full hypercube, while also allowing for efficient
10 sampling and log-density evaluations. This discussion will be added to the main document to improve its clarity. We
11 want to add that it is not obvious that eq. (6) is a density, but this follows from the proof of Proposition 1 by taking
12 $f = 1$ in eq. (9) therein. Fig. 1 and 2 shows that our proposed family is quite flexible and expressive while being
13 cheap to sample from. However, we will also try to include a more detailed numerical study of the proposed family of
14 densities. Eventually, the flexibility of the variational approximation can be increased using different complementary
15 work. We have illustrated in Section 6.2 that one can use a mixture of copula-like densities, that can also enhance
16 the flexibility of the marginal distribution. Similarly, one could use the new density within a semi-implicit variational
17 framework whose parameters are the output of a neural network conditional on some latent mixing variable.

18 **Predefined budget of dependency parameters (Reviewer #2):** Thank you for pointing out this very interesting question.
19 We think that this point is relevant but would require a lot of ideas and results which are out the scope of the document
20 and would distract the reader from the main ideas suggested in the paper.

21 **Clarification of the transformation \mathcal{H} (Reviewer #2 and #3):** The main reason to consider $U = \mathcal{H}(V)$ with
22 $V \sim c_\theta$ was 1) numerical stability since we need to compute quantile functions only on the interval $[\epsilon, 1 - \epsilon]$ using this
23 transformation 2) to increase the flexibility of our proposed family. We suggest to take initially at random $\delta \in [0, 1]^d$
24 for the transformation \mathcal{H} such that $\mathbb{P}(\delta_i = \epsilon) = p$ and $\mathbb{P}(\delta_i = 1 - \epsilon) = 1 - p$ with $p, \epsilon \in (0, 1)$ (in our experiments
25 $\epsilon = 0.01$ and $p = 1/2$). We found that choosing a different (large enough) value of ϵ tends to yield no large difference,
26 as this choice will get balanced by a different value of the standard deviation of the Gaussian marginal transformation.
27 However, we observed that the parameter p can impact the representative power of the variational distribution and the
28 best and most sensible choice was $p = 1/2$ since it leads to a balanced proportion of components of δ equal to ϵ and
29 $1 - \epsilon$. A more detailed discussion will be added to the document on this point.

30 **Ablation studies and different normalizing flows (Reviewer #1 and #3):** We remark that using a Gaussian mean-
31 field distribution with rotations basically yields still a Gaussian approximation, assuming that the small effect of the
32 transformation \mathcal{H} can be neglected. For $X = \mathcal{O}X'$ with a mean-field distribution $X' \sim \mathcal{N}(\mu, \Lambda)$ and some rotation
33 matrix \mathcal{O} , we have $X \sim \mathcal{N}(\mathcal{O}\mu, \mathcal{O}\Lambda\mathcal{O}^\top)$. We have shown in sections 6.1 and 6.2 that using a copula-like density
34 instead of an independent-copula base distribution can be beneficial, as the latter corresponds to the results for the
35 full-rank Gaussian case with one rotation in dimension $d = 2$. Following the reviews, we have also performed an
36 analogous study for the BNNs with the basic message being that the transformation \mathcal{H} is essential for the copula-like
37 density, whereas application of additional rotations yields only a smaller improvement. Rotating a mean-field Gaussian
38 also performed less competitively. We will include more details in the revised version and for brevity refer to the
39 MNIST results in the next section. Replacing the rotations with non-linear transformations like Inverse Autoregressive
40 Flows would be an interesting idea to explore further, but we did not have the chance to explore this before the deadline.

41 **High-dimensional BNN and the rotation trick (Reviewer #1 and #3):** The variational density for the BNNs includes
42 the rotations. We have not stated this explicitly and this might have caused confusion as we also referred to the density
43 without the rotations as copula-like in the ablation study for the logistic regression. We will make the appropriate
44 correction in the revised document. For the considered BNN with 200k latent variables, the complexity increases by
45 a factor of around 13 compared to a mean-field model. We expect that more ad-hoc tricks can be used to adjust the
46 rotations to some computational budget. For instance, one could consider the series of sparse rotations $\mathcal{O}_1, \dots, \mathcal{O}_k$, but
47 with $2^k < d$, thereby allowing for rotations of the more adjacent latent variables only. We acknowledge that the format
48 and size Figure 3 is not suitable. We plan to illustrate our result using only one figure presenting $\text{Cor}(\text{vec}(W_{:,0}^3, W_{:,1}^3))$
49 (corresponding to Fig. 3(a)) with better colors and with not all the neurons. We will also present our results by making
50 bigger and more designed plots in the supplementary material. The motivation for the model in the last experiment was
51 to illustrate that the proposed approach can be used in high-dimensional structured Bayesian models without having to
52 specify more model-specific dependency assumptions in the variational family. Our experimental MNIST results show
53 that for this given model, a copula-like approach can perform better (error rate 1.70% with rotations and 1.78% without)
54 than a Gaussian family (3.4% with rotations but without \mathcal{H} ; 3.82% in the mean-field; a low-rank covariance structure
55 did not converge in our implementation). The proposed density seems a useful alternative to a Gaussian one for the
56 considered model and we have not analysed if it compares favourably in other models mentioned in Table 4, which has
57 been included to indicate that the prediction errors are roughly in line with current work for fully connected networks.