- We thank the reviewers for constructive comments and questions. 1
- One common major concern of the reviewers is whether EBF can be efficiently implemented. We have been thinking 2
- about this problem since the submission deadline and have made substantial progress in designing a practical version 3
- of EBF algorithm. To solve the optimization problem in line5 Algorithm1 efficiently, we relax some constraints (e.g. 4
- $\pi_k$  is deterministic), and apply EVI alike techniques to compute the constrainted optimal policy in the extended MDP. 5
- However, we have not finished the details completely before the rebuttal deadline. As a result, we have to give up 6
- adding the implementation of EBF algorithm to this submission. 7
- Other concerns are answered as below. The typos, wrong references, missing references and unclear expressions like 8
- Def. 4 would be fixed carefully in the final version (if possible): 9

## To reviewer1: 10

- 1. About intuition: we can catch more information about  $h^*$  from the history trajectory (See line1 Algorithm2). One 11
- important difference to previous methods is that, the order of samples in the trajectory matters in EBF algorithm, while 12
- previous methods only use  $N_{s,a}$  and  $N_{s,a,s'}$  to build confidence set. As a result,  $\mathcal{H}$  is a tighter confidence set for  $h^*$ , which enables us to prove ④ and part of ③ are lower order terms. (See Lemma5 and Appendix.C.5) 2. We do not bound  $|h_k h^*|_{\infty}$ . Indeed, it suffices to bound  $N_{s,a,s'}^{(t_k)}|\delta_{s,s'}^* \delta_{k,s,s'}|$  up to  $\tilde{O}(\sqrt{T})$  and this is exactly 13 14
- 15 what we do. 16
- 3. We will mention the literature which first uses Bernstein's inequality to bound the uncertainty. 17
- 18
- 4. An MDP is flat iff  $r_{s,a} + P_{s,a}^T h^* = h_s^* + \rho^*$  for any s, a. We use the notation  $reg_{s,a}$  because we regard it as a generalization of  $\Delta_a = \mu^* \mu_a$  in multi-armed bandit (MAB) problem. We will explain these expressions more 19 clearly. 20
- 5. As for the usage of Lemma1 in the proof of Theorem1, we apply Lemma1 to the virtual MDP with increased reward 21
- function. Although the original MDP M might not be flat, the new MDP is flat after increasing  $r_{s,a}$  by  $reg_{s,a}$ . (See 22 Appendix.C.5) 23
- 6. You mean REGAL.C rather than REGAL.D? We can search among the MDPs with constant gains so that the problem 24
- is well-posed, although it is intractable in practice. 25
- 7. The modified version of (2) refers to line1 in Algorithm2. 26
- 8. We are sorry that we are unaware of the state of the art. Consequently, we only improve an  $\sqrt{S}$  (or  $\sqrt{\Gamma}$ ) factor 27
- compared to the work you mentioned. Nevertheless, to out best of knowledge, this is the first upper bound which matches 28
- the lower bound with logarithm factors ignored. We will also mention [Talebi and Maillard, 2018]. In our analysis, 29

30 the dominant term in regret is 
$$\sum_{s,a} \sqrt{N_{s,a}^{(T)} V(P_{s,a}, h^*)} \le \sqrt{\sum_{s,a} N_{s,a}^{(T)} \sum_{s,a} V(P_{s,a}, h^*)} = \sqrt{T \sum_{s,a} V(P_{s,a}, h^*)},$$

which outperforms the result in [Talebi and Maillard, 2018] by at least an  $\sqrt{S}$  factor. 31

## To reviewer3: 32

- 1. We will add a reference of upper bound of  $\tilde{O}(\sqrt{N})$  in line 142. 33
- 2. We will check the related works section carefully. We will mention the works in the comments of reviewer1. The 34
- analysis about REGAL.C [Bartlett and Tewari 09] is correct, although that paper contains some other mistakes. 35

## To reviewer4: 36

- 1. You mean a problem-dependent regret bound of  $O(poly(S, A, H) \sum_{s,a} \log(T)/reg_{s,a})$  like the regret bound of 37
- $O(\sum_a \log(T)/\Delta_a)$  in the MAB problem? We can prove this regret bound is unreachable in the worst case where some 38
- state s has o(T) visit count. Under the assumption the visit count (in expectation) of each state s is at least CT for 39
- some conditional number C, our method for estimating the bias function works, and thus it is hopeful get a regret bound of  $O(poly(S, A, H, \frac{1}{C}) \sum_{s,a} \log(T)/reg_{s,a})$ . However, this assumption seems too strong for undiscountted RL. 40
- 41
- 2. In the case H is known, the example for lower bound of  $\Omega(\sqrt{SATH})$  was proposed in [Bartlett and Tewari, 2009], 42 although the authors claimed a wrong lower bound. 43
- 3. Yes. There is a similar mistake in the proof of Lemma3 [Osband and Van Roy 16]. 44
- 4. Yes. But regret bound of  $O(\sqrt{SATH})$  has been proved for finite horizon MDP with efficient algorithms (e.g. [Azar 45 et al.2017]. 46
- 5. We have sent an email to the authors, but did not get replies before the rebuttal deadline. We ensure that this issue 47
- will be carefully dealt with according to their suggestion. 48