Deep Reinforcement Learning through Policy Optimization

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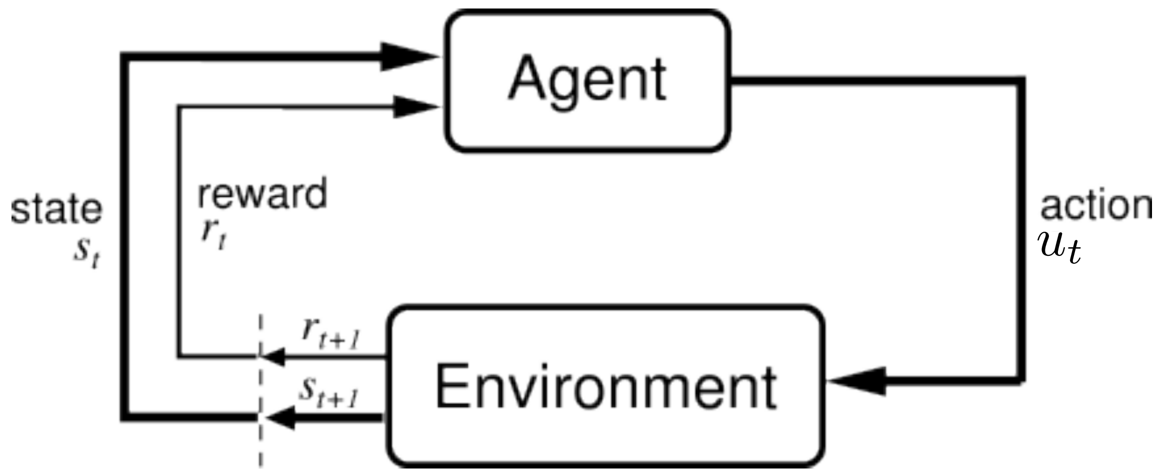
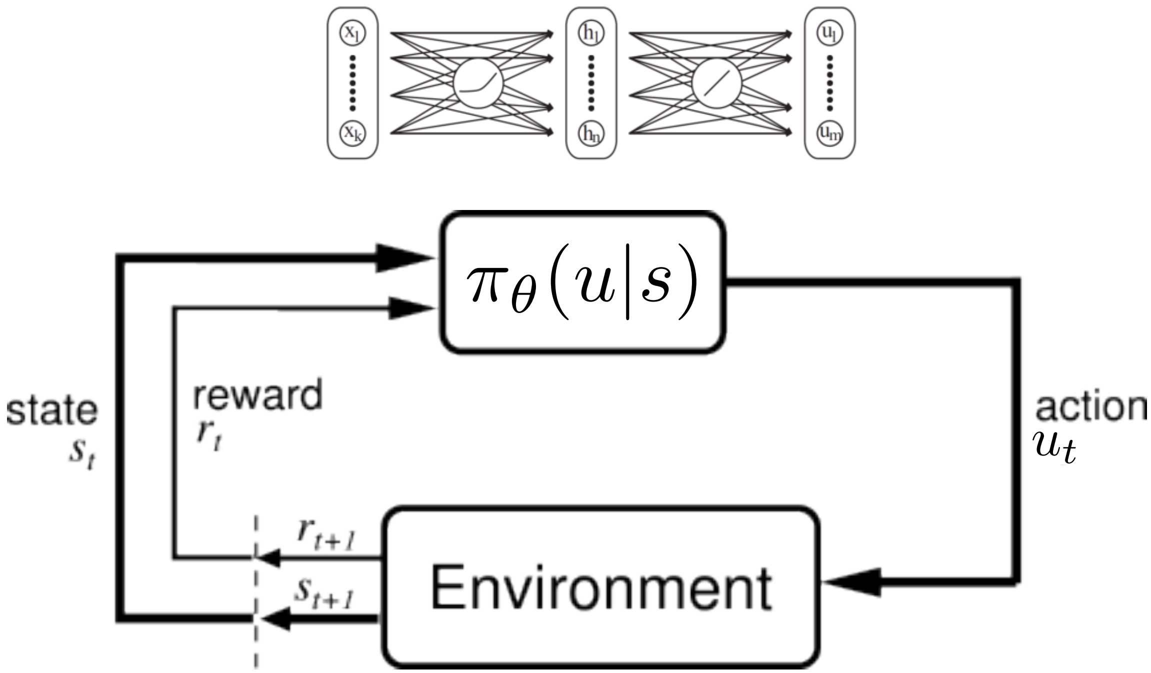
Reinforcement Learning



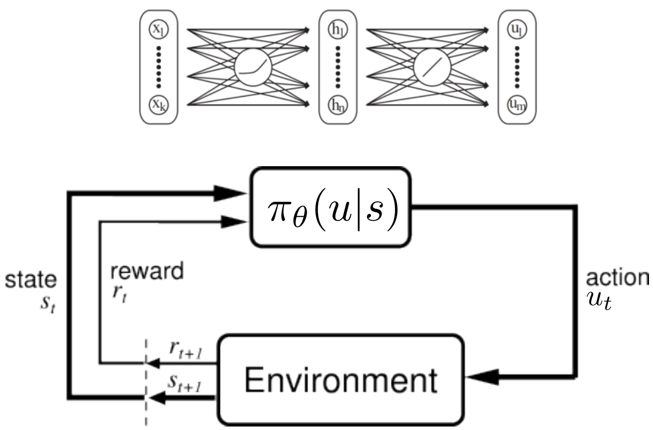
[Figure source: Sutton & Barto, 1998]

Policy Optimization



[Figure source: Sutton & Barto, 1998]

Policy Optimization



- Consider control policy parameterized by parameter vector θ

$$\max_{\theta} \quad \mathrm{E}[\sum_{t=0}^{H} R(s_t)|\pi_\theta]$$

- Often stochastic policy class (smooths out the problem):

  $\pi_\theta(u|s)$: probability of action u in state s

[Figure source: Sutton & Barto, 1998]

Why Policy Optimization

- Often π can be simpler than Q or V
  - E.g., robotic grasp
- V: doesn't prescribe actions
  - Would need dynamics model (+ compute 1 Bellman back-up)
- Q: need to be able to efficiently solve $\arg\max_u Q_\theta(s, u)$
  - Challenge for continuous / high-dimensional action spaces*

*some recent work (partially) addressing this:

NAF: Gu, Lillicrap, Sutskever, Levine ICML 2016

Input Convex NNs: Amos, Xu, Kolter arXiv 2016

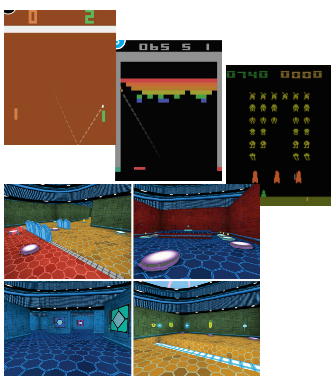
Example Policy Optimization Success Stories

Kohl and Stone, 2004

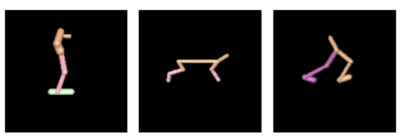
Ng et al, 2004

Tedrake et al, 2005

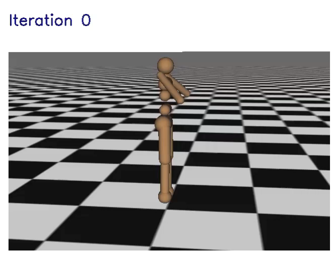
Kober and Peters, 2009



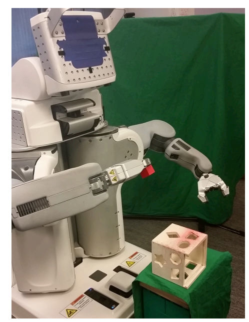
Mnih et al, 2015
(A3C)



Silver et al, 2014
(DPG)
Lillicrap et al, 2015
(DDPG)



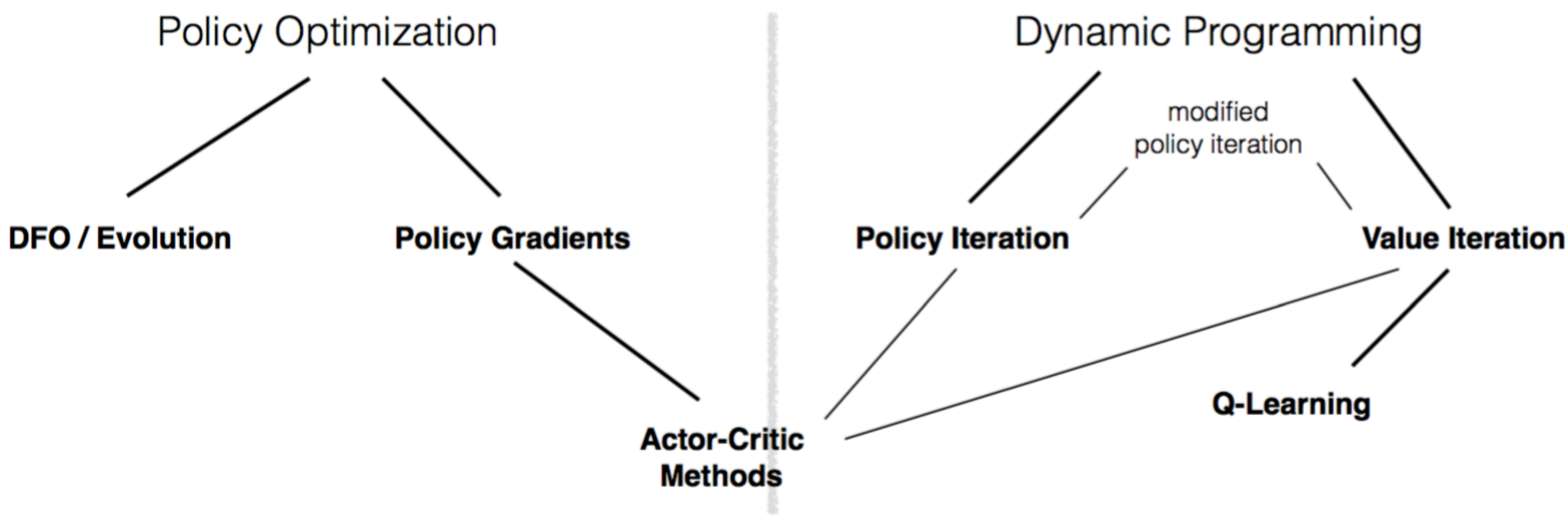
Schulman et al, 2016 (TRPO + GAE)



Levine*, Finn*, et al, 2016
(GPS)

Silver*, Huang*, et al, 2016
(AlphaGo**)

Policy Optimization in the RL Landscape



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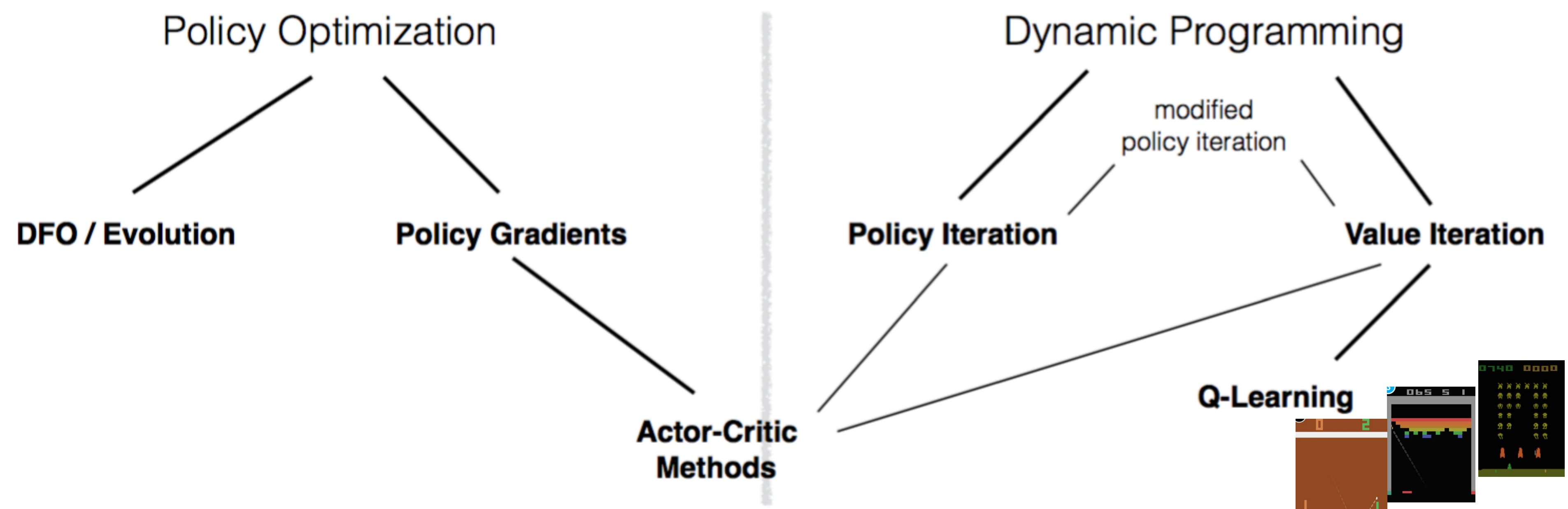
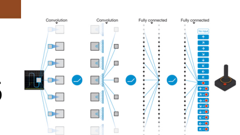


DQN: Mnih et al, Nature 2015
Double DQN: Van Hasselt et al, AAAI 2015
Dueling Architecture: Wang et al, ICML 2016
Prioritized Replay: Schaul et al, ICLR 2016
David Silver ICML 2016 tutorial



Outline

- Derivative free methods
  - Cross Entropy Method (CEM) / Finite Differences / Fixing Random Seed
- Likelihood Ratio (LR) Policy Gradient
  - Derivation / Connection w/Importance Sampling
- Natural Gradient / Trust Regions (-> TRPO)
- Variance Reduction using Value Functions (Actor-Critic) (-> GAE, A3C)
- Pathwise Derivatives (PD) (-> DPG, DDPG, SVG)
- Stochastic Computation Graphs (generalizes LR / PD)
- Guided Policy Search (GPS)
- Inverse Reinforcement Learning

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Cross-Entropy Method

$$\max_\theta \ U(\theta) = \max_\theta \ \mathrm{E}[\sum_{t=0}^{H} R(s_t)|\pi_\theta]$$

- Views *U* as a black box
- Ignores all other information other than *U* collected during episode

= evolutionary algorithm

population: $P_{\mu^{(i)}}(\theta)$

<u>CEM:</u>

for iter i = 1, 2, …
  for population member e = 1, 2, …
    sample $\theta^{(e)} \sim P_{\mu^{(i)}}(\theta)$
    execute roll-outs under $\pi_{\theta^{(e)}}$
    store $(\theta^{(e)}, U(e))$
  endfor
  $\mu^{(i+1)} = \arg\max_\mu \sum_{\bar{e}} \log P_\mu(\theta^{(\bar{e})})$
    where $\bar{e}$ indexes over top p %
endfor

Cross-Entropy Method

- Can work embarrassingly well

Method	Mean Score	Reference
Nonreinforcement learning		
Hand-coded	631,167	Dellacherie (Fahey, 2003)
Genetic algorithm	586,103	(Böhm et al., 2004)
Reinforcement learning		
Relational reinforcement learning+kernel-based regression	$\approx$50	Ramon and Driessens (2004)
Policy iteration	3183	Bertsekas and Tsitsiklis (1996)
Least squares policy iteration	<3000	Lagoudakis, Parr, and Littman (2002)
Linear programming + Bootstrap	4274	Farias and van Roy (2006)
Natural policy gradient	$\approx$6800	Kakade (2001)
CE+RL	21,252	
CE+RL, constant noise	72,705	
CE+RL, decreasing noise	348,895	

István Szita and András Lörincz. "Learning Tetris using the noisy cross-entropy method". In: *Neural computation* 18.12 (2006), pp. 2936–2941

Approximate Dynamic Programming Finally Performs Well in the Game of Tetris

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Team Maia, FRANCE
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[NIPS 2013]

Closely Related Approaches

CEM:

for iter i = 1, 2, …

  for population member e = 1, 2, …

    sample $\theta^{(e)} \sim P_{\mu^{(i)}}(\theta)$

    execute roll-outs under $\pi_{\theta^{(e)}}$

    store $(\theta^{(e)}, U(e))$

  endfor

$$\boxed{\mu^{(i+1)} = \arg\max_{\mu} \sum_{\bar{e}} \log P_{\mu}(\theta^{(\bar{e})}) \quad \text{where } \bar{e} \text{ indexes over top p \%}}$$

endfor

- Reward Weighted Regression (RWR)
  - Dayan & Hinton, NC 1997; Peters & Schaal, ICML 2007

$$\boxed{\mu^{(i+1)} = \arg\max_{\mu} \sum_{e} q(U(e), P_{\mu}(\theta^{(e)})) \log P_{\mu}(\theta^{(e)})}$$

- Policy Improvement with Path Integrals (PI^2)
  - PI2: Theodorou, Buchli, Schaal JMLR2010; Kappen, 2007; (PI2-CMA: Stulp & Sigaud ICML2012)

$$\boxed{\mu^{(i+1)} = \arg\max_{\mu} \sum_{e} \exp(\lambda U(e)) \log P_{\mu}(\theta^{(e)})}$$

- Covariance Matrix Adaptation Evolutionary Strategy (CMA-ES)
  - CMA: Hansen & Ostermeier 1996; (CMA-ES: Hansen, Muller, Koumoutsakos 2003)

$$\boxed{(\mu^{(i+1)}, \Sigma^{(i+1)}) = \arg\max_{\mu,\Sigma} \sum_{\bar{e}} w(U(\bar{e})) \log \mathcal{N}(\theta^{(\bar{e})}; \mu, \Sigma)}$$

- PoWER
  - Kober & Peters, NIPS 2007 (also applies importance sampling for sample re-use)

$$\boxed{\mu^{(i+1)} = \mu^{(i)} + \left(\sum_{e} (\theta^{(e)} - \mu^{(i)}) U(e)\right) / \left(\sum_{e} U(e)\right)}$$

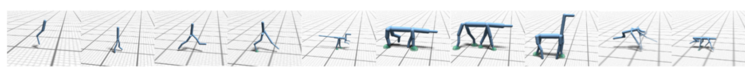
Applications

Covariance Matrix Adaptation (CMA) has become standard in graphics [Hansen, Ostermeier, 1996]

Optimal Gait and Form for Animal Locomotion

Kevin Wampler* Zoran Popović

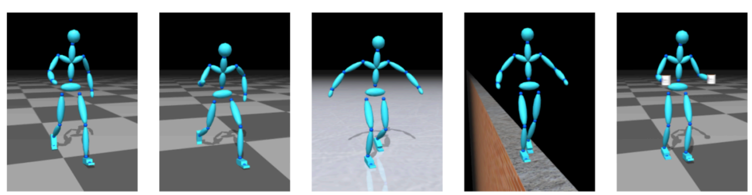
University of Washington



Optimizing Walking Controllers for Uncertain Inputs and Environments

Jack M. Wang David J. Fleet Aaron Hertzmann

University of Toronto



PoWER [Kober&Peters, MLJ 2011]

Cross-Entropy / Evolutionary Methods

- Full episode evaluation, parameter perturbation
- Simple
- Main caveat: best when number of parameters is relatively small
  - i.e., number of population members comparable to or larger than number of (effective) parameters

  → in practice OK if low-dimensional θ and willing to do do many runs

  → Easy-to-implement baseline, great for comparisons!

Black Box Gradient Computation

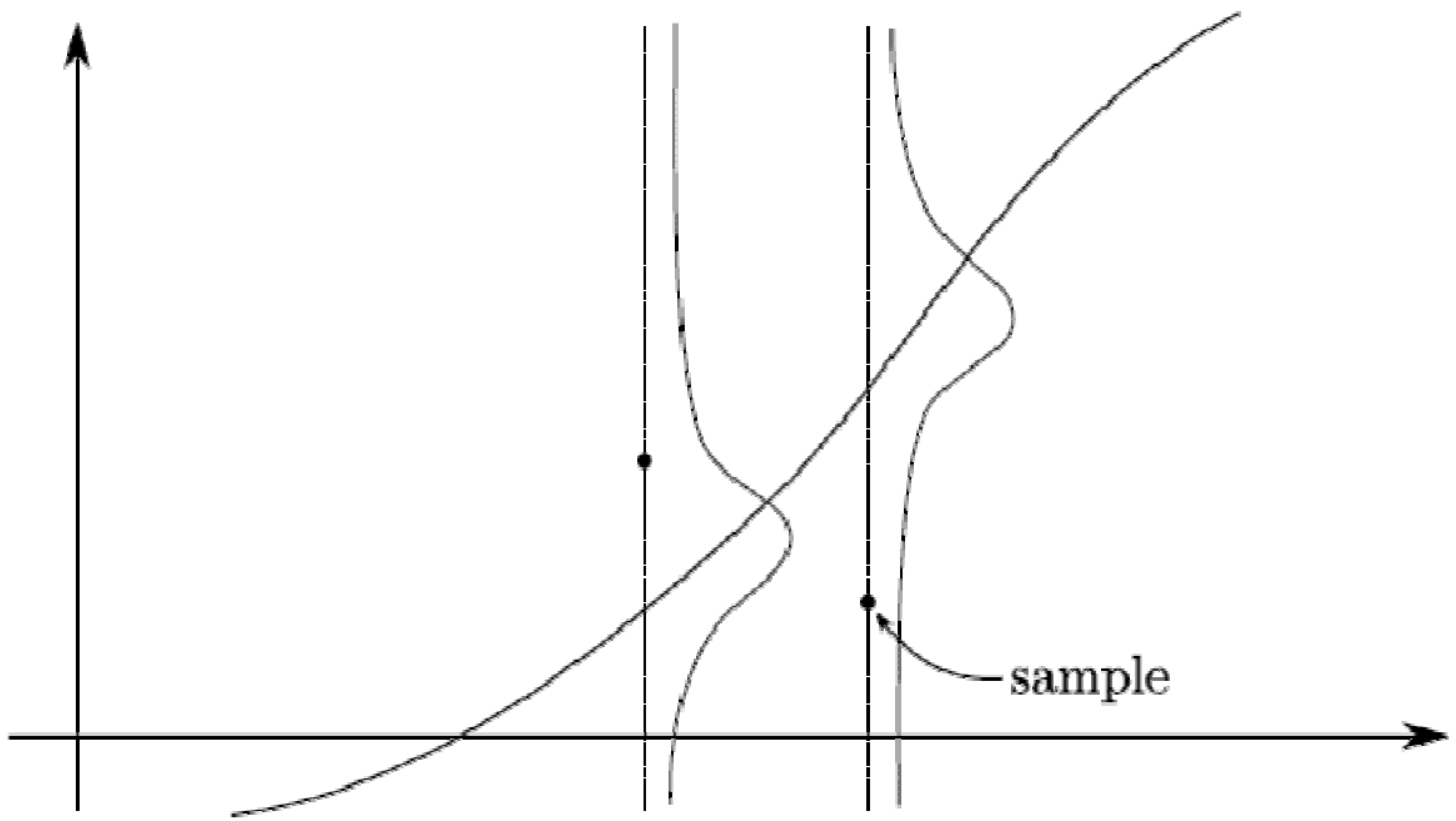
We can compute the gradient g using standard finite difference methods, as follows:

$$\frac{\partial U}{\partial \theta_j}(\theta) = \frac{U(\theta + \epsilon e_j) - U(\theta - \epsilon e_j)}{2\epsilon}$$

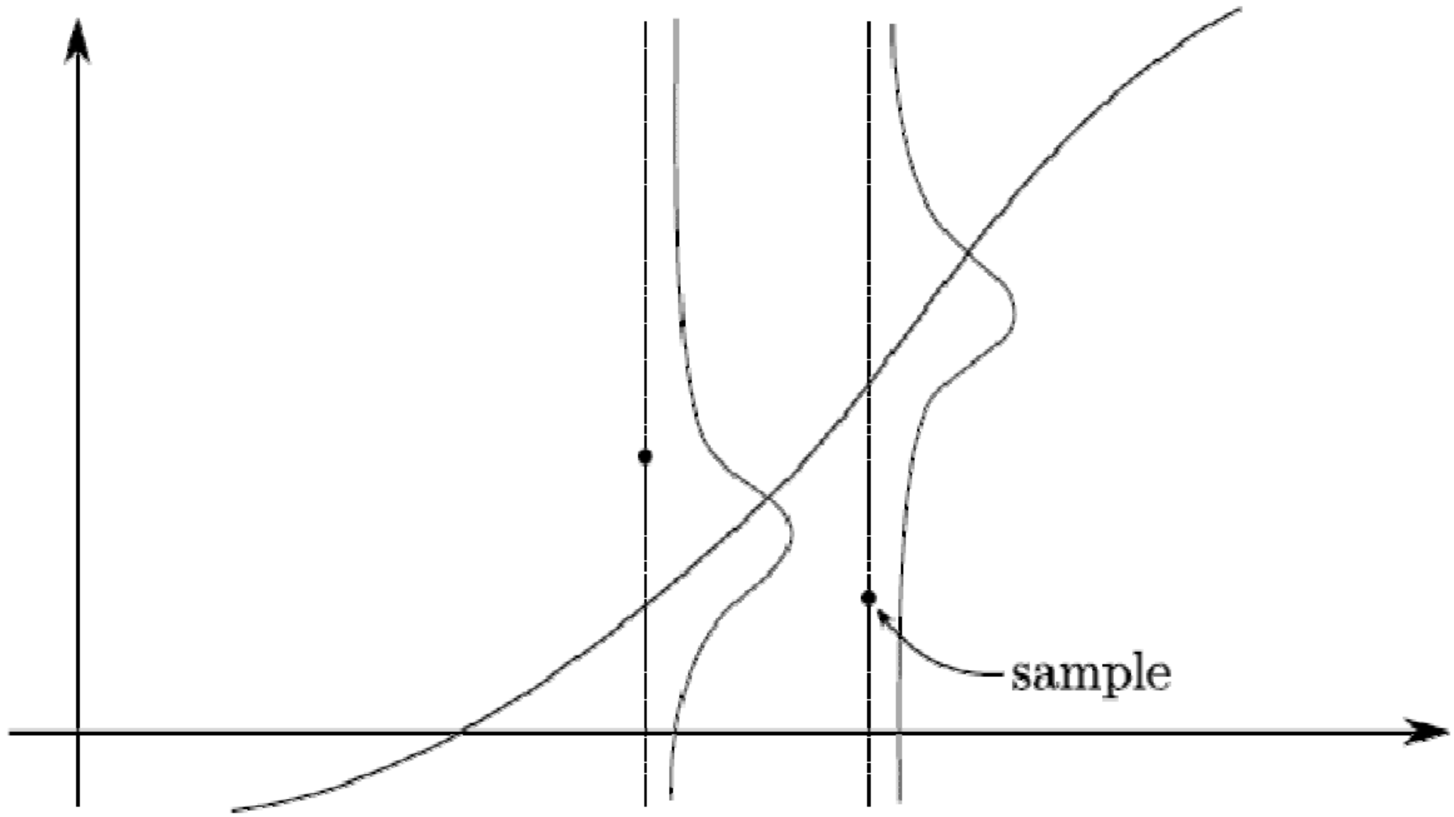
Where:

$$e_j = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \begin{matrix} \\ \\ \\ \\ \leftarrow \; j\text{'th entry} \\ \\ \\ \\ \end{matrix}$$

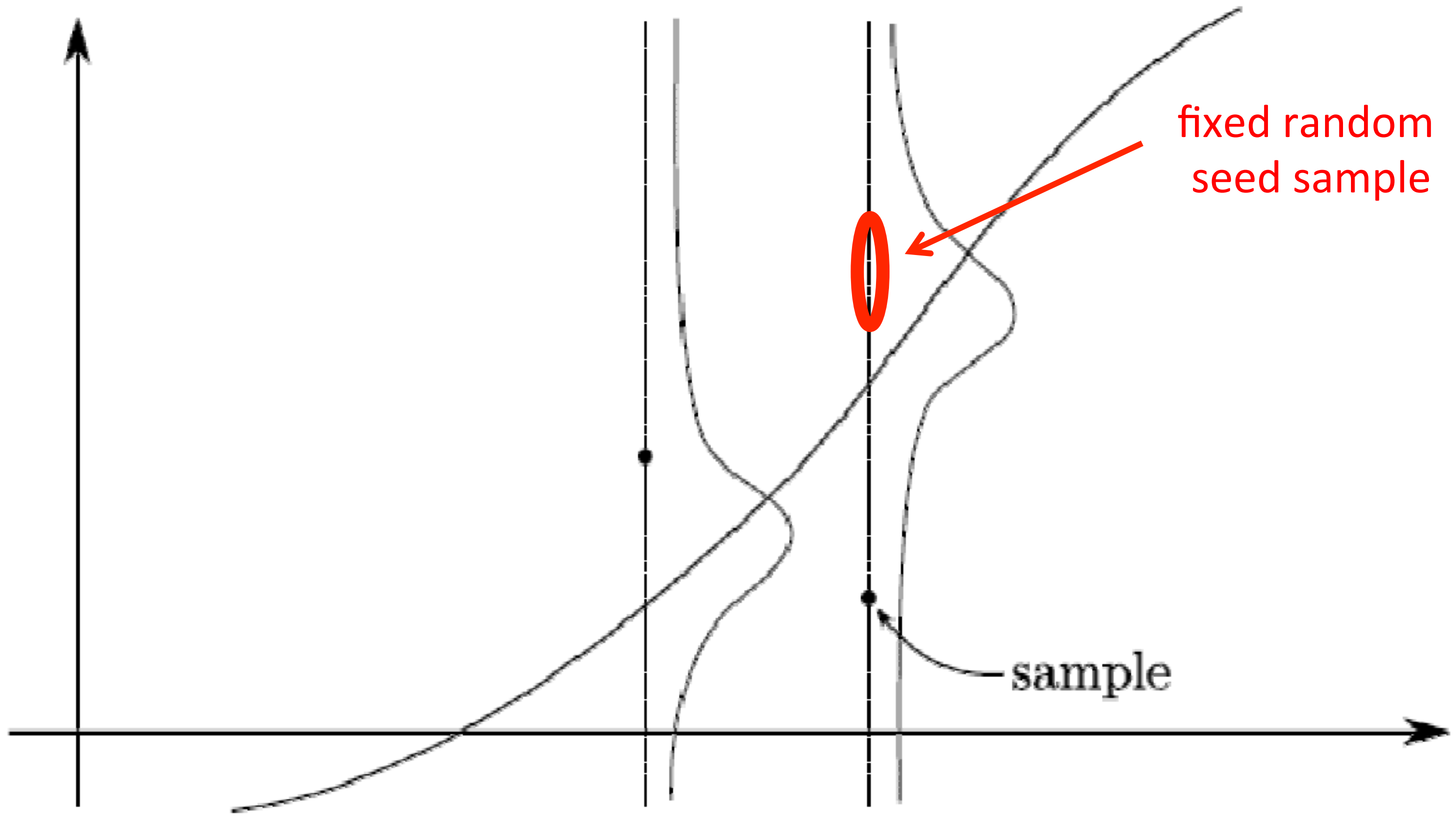
Challenge: Noise Can Dominate



Solution 1: Average over many samples



Solution 2: Fix random seed



Solution 2: Fix random seed

- Randomness in policy and dynamics
  - But can often only control randomness in policy..

- Example: wind influence on a helicopter is stochastic, but if we assume the same wind pattern across trials, this will make the different choices of θ more readily comparable

- *Note: equally applicable to evolutionary methods*

[Ng & Jordan, 2000] provide theoretical analysis of gains from fixing randomness ("pegasus")

[Policy search was done in simulation] [Ng + al, ISER 2004]

Learning to Hover

x, y, z: x points forward along the helicopter, y sideways to the right, z downward.

n_x, n_y, n_z: rotation vector that brings helicopter back to "level" position (expressed in the helicopter frame).

$$u_{collective} = \theta_1 \cdot f_1(z^* - z) + \theta_2 \cdot \dot{z}$$

$$u_{elevator} = \theta_3 \cdot f_2(x^* - x) + \theta_4 f_4(\dot{x}) + \theta_5 \cdot q + \theta_6 \cdot n_y$$

$$u_{aileron} = \theta_7 \cdot f_3(y^* - y) + \theta_8 f_5(\dot{y}) + \theta_9 \cdot p + \theta_{10} \cdot n_x$$

$$u_{rudder} = \theta_{11} \cdot r + \theta_{12} \cdot n_z$$

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Likelihood Ratio Policy Gradient

We let τ denote a state-action sequence $s_0, u_0, \ldots, s_H, u_H$. We overload notation: $R(\tau) = \sum_{t=0}^{H} R(s_t, u_t)$.

$$U(\theta) = \mathrm{E}[\sum_{t=0}^{H} R(s_t, u_t); \pi_\theta] = \sum_{\tau} P(\tau; \theta) R(\tau)$$

In our new notation, our goal is to find θ:

$$\max_{\theta} U(\theta) = \max_{\theta} \sum_{\tau} P(\tau; \theta) R(\tau)$$

Likelihood Ratio Policy Gradient

$$U(\theta) = \sum_{\tau} P(\tau;\theta) R(\tau)$$

Taking the gradient w.r.t. θ gives

$$\nabla_\theta U(\theta) = \nabla_\theta \sum_{\tau} P(\tau;\theta) R(\tau)$$

[Aleksandrov, Sysoyev, & Shemeneva, 1968]
[Rubinstein, 1969]
[Glynn, 1986]
[Reinforce, Williams 1992]
[GPOMDP, Baxter & Bartlett, 2001]

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\end{aligned}$$

Approximate with the empirical estimate for m sample paths under policy π_θ:

$$\nabla_\theta U(\theta) \approx \hat{g} = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta \log P(\tau^{(i)};\theta) R(\tau^{(i)})$$

[Aleksandrov, Sysoyev, & Shemeneva, 1968]
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Derivation from Importance Sampling

$$U(\theta) = \mathbb{E}_{\tau \sim \theta_{\text{old}}} \left[\frac{P(\tau|\theta)}{P(\tau|\theta_{\text{old}})} R(\tau) \right]$$

[Tang&Abbeel, NIPS 2011]

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$$= \mathbb{E}_{\tau \sim \theta_{\text{old}}} \left[\nabla_\theta \log \left. P(\tau|\theta) \right|_{\theta_{\text{old}}} R(\tau) \right]$$

Suggests we can also look at more than just gradient!
E.g., can use importance sampled objective as "surrogate loss" (locally)

[Tang&Abbeel, NIPS 2011]

Likelihood Ratio Gradient: Validity

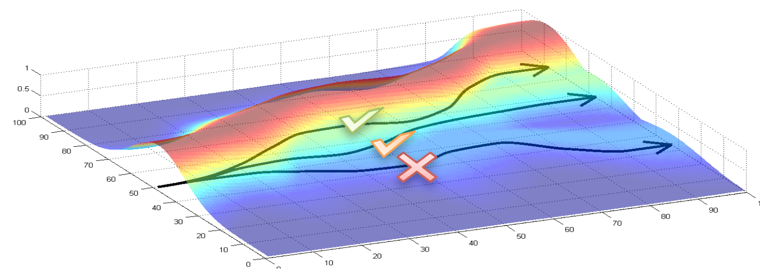
$$\nabla U(\theta) \approx \hat{g} = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta \log P(\tau^{(i)}; \theta) R(\tau^{(i)})$$

- Valid even if R is discontinuous, and unknown, or sample space (of paths) is a discrete set

Likelihood Ratio Gradient: Intuition

$$\nabla U(\theta) \approx \hat{g} = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta \log P(\tau^{(i)}; \theta) R(\tau^{(i)})$$

- Gradient tries to:
  - Increase probability of paths with positive R
  - Decrease probability of paths with negative R



! Likelihood ratio changes probabilities of experienced paths, does not try to change the paths (see Path Derivative later)

Let's Decompose Path into States and Actions

$$\nabla_\theta \log P(\tau^{(i)};\theta) = \nabla_\theta \log \left[\prod_{t=0}^{H} \underbrace{P(s_{t+1}^{(i)}|s_t^{(i)}, u_t^{(i)})}_{\text{dynamics model}} \cdot \underbrace{\pi_\theta(u_t^{(i)}|s_t^{(i)})}_{\text{policy}} \right]$$

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$$= \nabla_\theta \left[\sum_{t=0}^{H} \log P(s_{t+1}^{(i)}|s_t^{(i)},u_t^{(i)}) + \sum_{t=0}^{H} \log \pi_\theta(u_t^{(i)}|s_t^{(i)}) \right]$$

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$$= \nabla_\theta \sum_{t=0}^{H} \log \pi_\theta(u_t^{(i)}|s_t^{(i)})$$

$$= \sum_{t=0}^{H} \underbrace{\nabla_\theta \log \pi_\theta(u_t^{(i)}|s_t^{(i)})}_{\text{no dynamics model required!!}}$$

Likelihood Ratio Gradient Estimate

The following expression provides us with an unbiased estimate of the gradient, and we can compute it without access to a dynamics model:

$$\hat{g} = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta \log P(\tau^{(i)}; \theta) R(\tau^{(i)})$$

Here:

$$\nabla_\theta \log P(\tau^{(i)}; \theta) = \sum_{t=0}^{H} \underbrace{\nabla_\theta \log \pi_\theta(u_t^{(i)} | s_t^{(i)})}_{\text{no dynamics model required!!}}$$

Unbiased means:

$$\mathrm{E}[\hat{g}] = \nabla_\theta U(\theta)$$

Likelihood Ratio Gradient Estimate

- As formulated thus far: unbiased but very noisy
- Fixes that lead to real-world practicality
  - Baseline
  - Temporal structure

  - Also: KL-divergence trust region / natural gradient (= general trick, equally applicable to perturbation analysis and finite differences)

Likelihood Ratio Gradient: Baseline

$$\nabla U(\theta) \approx \hat{g} = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta \log P(\tau^{(i)}; \theta) R(\tau^{(i)})$$

- To build intuition, let's assume R > 0
  - Then tries to increase probabilities of all paths

→ Consider baseline b:

$$\nabla U(\theta) \approx \hat{g} = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta \log P(\tau^{(i)}; \theta)(R(\tau^{(i)}) - b)$$

still unbiased

[Williams 1992]

$$\begin{aligned}
&\mathbb{E}\left[\nabla_\theta \log P(\tau;\theta) b\right] \\
=&\sum_\tau P(\tau;\theta) \nabla_\theta \log P(\tau;\theta) b \\
=&\sum_\tau P(\tau;\theta) \frac{\nabla_\theta P(\tau;\theta)}{P(\tau;\theta)} b \\
=&\sum_\tau \nabla_\theta P(\tau;\theta) b \\
=&\nabla_\theta \left(\sum_\tau P(\tau) b \right) \\
=&\nabla_\theta \left(b\right) \\
=&0
\end{aligned}$$

Good choices for b?

$$b = \mathbb{E}\left[R(\tau)\right] \approx \frac{1}{m} \sum_{i=1}^{m} R(\tau^{(i)})$$

$$b = \frac{\sum_i \left(\nabla_\theta \log P(\tau^{(i)};\theta)\right)^2 R(\tau^{(i)})}{\sum_i \left(\nabla_\theta \log P(\tau^{(i)};\theta)\right)^2}$$

[See: Greensmith, Bartlett, Baxter, JMLR 2004 for variance reduction techniques.]

Likelihood Ratio and Temporal Structure

- Current estimate: $$\hat{g} = \frac{1}{m}\sum_{i=1}^{m} \nabla_\theta \log P(\tau^{(i)};\theta)(R(\tau^{(i)}) - b)$$ $$= \frac{1}{m}\sum_{i=1}^{m} \left(\sum_{t=0}^{H-1} \nabla_\theta \log \pi_\theta(u_t^{(i)}|s_t^{(i)})\right)\left(\sum_{t=0}^{H-1} R(s_t^{(i)}, u_t^{(i)}) - b\right)$$
- Future actions do not depend on past rewards, hence can lower variance by instead using: $$\frac{1}{m}\sum_{i=1}^{m}\sum_{t=0}^{H-1} \nabla_\theta \log \pi_\theta(u_t^{(i)}|s_t^{(i)}) \left(\sum_{k=t}^{H-1} R(s_k^{(i)}, u_k^{(i)}) - b(s_k^{(i)})\right)$$
- Good choice for b?

Expected return: $b(s_t) = \mathbb{E}\left[r_t + r_{t+1} + r_{t+2} + \ldots + r_{H-1}\right]$

→ Increase logprob of action proportionally to how much its returns are better than the expected return under the current policy

[Policy Gradient Theorem: Sutton et al, NIPS 1999; GPOMDP: Bartlett & Baxter, JAIR 2001; Survey: Peters & Schaal, IROS 2006]

Pseudo-code Reinforce aka Vanilla Policy Gradient

Algorithm 1 "Vanilla" policy gradient algorithm

Initialize policy parameter θ, baseline b
for iteration$=1, 2, \dots$ **do**
  Collect a set of trajectories by executing the current policy
  At each timestep in each trajectory, compute
   the *return* $R_t = \sum_{t'=t}^{T-1} \gamma^{t'-t} r_{t'}$, and
   the *advantage estimate* $\hat{A}_t = R_t - b(s_t)$.
  Re-fit the baseline, by minimizing $\|b(s_t) - R_t\|^2$,
   summed over all trajectories and timesteps.
  Update the policy, using a policy gradient estimate $\hat{g}$,
   which is a sum of terms $\nabla_\theta \log \pi(a_t \mid s_t, \theta)\hat{A}_t$
end for

~ [Williams, 1992]

Outline

- Derivative free methods
  - Cross Entropy Method (CEM) / Finite Differences / Fixing Random Seed
- Likelihood Ratio (LR) Policy Gradient
  - Derivation / Connection w/Importance Sampling
- ***Natural Gradient / Trust Regions (-> TRPO)***
- ***Variance Reduction using Value Functions (Actor-Critic) (-> GAE, A3C)***
- ***Pathwise Derivatives (PD) (-> DPG, DDPG, SVG)***
- ***Stochastic Computation Graphs (generalizes LR / PD)***
- Guided Policy Search (GPS)
- Inverse Reinforcement Learning

Trust Region Policy Optimization

Desiderata

Desiderata for policy optimization method:

- Stable, monotonic improvement. (How to choose stepsizes?)
- Good sample efficiency

Step Sizes

Why are step sizes a big deal in RL?

- Supervised learning
  - Step too far $\rightarrow$ next updates will fix it
- Reinforcement learning
  - Step too far $\rightarrow$ bad policy
  - Next batch: collected under bad policy
  - Can't recover, collapse in performance!

Surrogate Objective

- Let $\eta(\pi)$ denote the expected return of π
- We collect data with π_{old}. Want to optimize some objective to get a new policy π
- Define $L_{\pi_{\text{old}}}(\pi)$ to be the "surrogate objective"[1]

$$L(\pi) = \mathbb{E}_{\pi_{\text{old}}}\left[\frac{\pi(a \mid s)}{\pi_{\text{old}}(a \mid s)} A^{\pi_{\text{old}}}(s, a)\right]$$

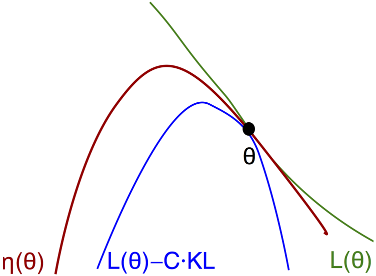
$$\nabla_\theta L(\pi_\theta)\big|_{\theta_{\text{old}}} = \nabla_\theta \eta(\pi_\theta)\big|_{\theta_{\text{old}}} \quad \text{(policy gradient)}$$

- Local approximation to the performance of the policy; does not depend on parameterization of π

[1] S. Kakade and J. Langford. "Approximately optimal approximate reinforcement learning". In: *ICML*. vol. 2. 2002, pp. 267–274.

Improvement Theory

- Theory: bound the difference between $L_{\pi_{\text{old}}}(\pi)$ and $\eta(\pi)$, the performance of the policy
- Result: $\eta(\pi) \geq L_{\pi_{\text{old}}}(\pi) - C \cdot \max_s \mathrm{KL}[\pi_{\text{old}}(\cdot \mid s), \pi(\cdot \mid s)]$, where $c = 2\epsilon\gamma/(1-\gamma)^2$
- Monotonic improvement guaranteed (MM algorithm)



Practical Algorithm: TRPO

- Constrained optimization problem

$$\max_{\pi} L(\pi), \text{ subject to } \overline{\mathrm{KL}}[\pi_{\mathrm{old}}, \pi] \leq \delta$$

$$\text{where } L(\pi) = \mathbb{E}_{\pi_{\mathrm{old}}}\left[\frac{\pi(a \mid s)}{\pi_{\mathrm{old}}(a \mid s)} A^{\pi_{\mathrm{old}}}(s, a)\right]$$

- Construct loss from empirical data

$$\hat{L}(\pi) = \sum_{n=1}^{N} \frac{\pi(a_n \mid s_n)}{\pi_{\mathrm{old}}(a_n \mid s_n)} \hat{A}_n$$

- Make quadratic approximation and solve with conjugate gradient algorithm

---

J. Schulman, S. Levine, P. Moritz, et al. "Trust Region Policy Optimization". In: *ICML*. 2015

Practical Algorithm: TRPO

for iteration$=1, 2, \ldots$ **do**

Run policy for T timesteps or N trajectories

Estimate advantage function at all timesteps

Compute policy gradient g

Use CG (with Hessian-vector products) to compute $F^{-1}g$

Do line search on surrogate loss and KL constraint

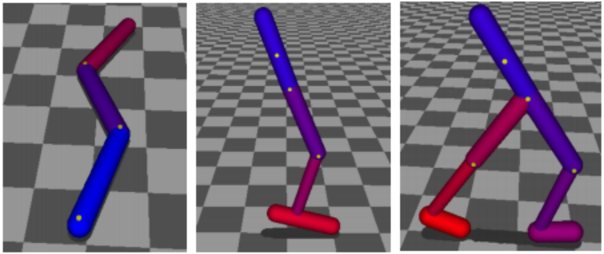
end for

J. Schulman, S. Levine, P. Moritz, et al. "Trust Region Policy Optimization". In: *ICML*. 2015

Practical Algorithm: TRPO

Applied to

- Locomotion controllers in 2D



- Atari games with pixel input

J. Schulman, S. Levine, P. Moritz, et al. "Trust Region Policy Optimization". In: *ICML*. 2015

"Proximal" Policy Optimization

- Use penalty instead of constraint

$$\underset{\theta}{\text{minimize}} \sum_{n=1}^{N} \frac{\pi_\theta(a_n \mid s_n)}{\pi_{\theta_{\text{old}}}(a_n \mid s_n)} \hat{A}_n - \beta \overline{\mathrm{KL}}[\pi_{\theta_{\text{old}}}, \pi_\theta]$$

"Proximal" Policy Optimization

- Use penalty instead of constraint

$$\underset{\theta}{\text{minimize}} \sum_{n=1}^{N} \frac{\pi_\theta(a_n \mid s_n)}{\pi_{\theta_{\text{old}}}(a_n \mid s_n)} \hat{A}_n - \beta \overline{\mathrm{KL}}[\pi_{\theta_{\text{old}}}, \pi_\theta]$$

- Pseudocode:

for iteration$=1, 2, \ldots$ **do**
    Run policy for T timesteps or N trajectories
    Estimate advantage function at all timesteps
    Do SGD on above objective for some number of epochs
    If KL too high, increase β. If KL too low, decrease β.
end for

“Proximal” Policy Optimization

- Use penalty instead of constraint

$$\underset{\theta}{\text{minimize}} \sum_{n=1}^{N} \frac{\pi_\theta(a_n \mid s_n)}{\pi_{\theta_{\text{old}}}(a_n \mid s_n)} \hat{A}_n - \beta \overline{\mathrm{KL}}[\pi_{\theta_{\text{old}}}, \pi_\theta]$$

- Pseudocode:

  for iteration$=1, 2, \ldots$ **do**

  Run policy for T timesteps or N trajectories

  Estimate advantage function at all timesteps

  Do SGD on above objective for some number of epochs

  If KL too high, increase β. If KL too low, decrease β.

  end for

- $\approx$ same performance as TRPO, but only first-order optimization

Variance Reduction Using Value Functions

Variance Reduction

- Now, we have the following policy gradient formula:

$$\nabla_\theta \mathbb{E}_\tau [R] = \mathbb{E}_\tau \left[\sum_{t=0}^{T-1} \nabla_\theta \log \pi(a_t \mid s_t, \theta) A^\pi(s_t, a_t) \right]$$

- A^π is not known, but we can plug in $\hat{A}_t$, an *advantage estimator*
- Previously, we showed that taking

$$\hat{A}_t = r_t + r_{t+1} + r_{t+2} + \cdots - b(s_t)$$

for any function $b(s_t)$, gives an unbiased policy gradient estimator.
$b(s_t) \approx V^\pi(s_t)$ gives variance reduction.

The Delayed Reward Problem

- With policy gradient methods, we are confounding the effect of multiple actions:

$$\hat{A}_t = r_t + r_{t+1} + r_{t+2} + \cdots - b(s_t)$$

  mixes effect of $a_t, a_{t+1}, a_{t+2}, \ldots$
- SNR of $\hat{A}_t$ scales roughly as $1/T$
  - Only a_t contributes to *signal* $A^\pi(s_t, a_t)$, but $a_{t+1}, a_{t+2}, \ldots$ contribute to noise.

Variance Reduction with Discounts

- Discount factor γ, $0 < \gamma < 1$, downweights the effect of rewars that are far in the future—ignore long term dependencies
- We can form an advantage estimator using the *discounted return*:

$$\hat{A}_t^{\gamma} = \underbrace{r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots}_{\text{discounted return}} - b(s_t)$$

  reduces to our previous estimator when $\gamma = 1$.
- So advantage has expectation zero, we should fit baseline to be *discounted value function*

$$V^{\pi,\gamma}(s) = \mathbb{E}_{\tau}\left[r_0 + \gamma r_1 + \gamma^2 r_2 + \ldots \mid s_0 = s\right]$$

- Discount γ is similar to using a horizon of $1/(1-\gamma)$ timesteps
- $\hat{A}_t^{\gamma}$ is a biased estimator of the advantage function

Value Functions in the Future

- Baseline accounts for and removes the effect of *past* actions
- Can also use the value function to estimate future rewards

$$
\begin{aligned}
& r_t + \gamma V(s_{t+1}) && \text{cut off at one timestep} \\
& r_t + \gamma r_{t+1} + \gamma^2 V(s_{t+2}) && \text{cut off at two timesteps} \\
& \ldots && \\
& r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots && \infty \text{ timesteps (no } V\text{)}
\end{aligned}
$$

Value Functions in the Future

- Subtracting out baselines, we get advantage estimators

$$
\begin{aligned}
\hat{A}_t^{(1)} &= r_t + \gamma V(s_{t+1}) - V(s_t) \\
\hat{A}_t^{(2)} &= r_t + r_{t+1} + \gamma^2 V(s_{t+2}) - V(s_t) \\
&\dots \\
\hat{A}_t^{(\infty)} &= r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \dots - V(s_t)
\end{aligned}
$$

- $\hat{A}_t^{(1)}$ has low variance but high bias, $\hat{A}_t^{(\infty)}$ has high variance but low bias.
- Using intermediate k (say, 20) gives an intermediate amount of bias and variance

Finite-Horizon Methods: Advantage Actor-Critic

- A2C / A3C uses this fixed-horizon advantage estimator

V. Mnih, A. P. Badia, M. Mirza, et al. "Asynchronous Methods for Deep Reinforcement Learning". In: *ICML* (2016)

Finite-Horizon Methods: Advantage Actor-Critic

- A2C / A3C uses this fixed-horizon advantage estimator
- Pseudocode

for iteration$=1, 2, \ldots$ **do**
  Agent acts for T timesteps (e.g., $T = 20$),
  For each timestep t, compute

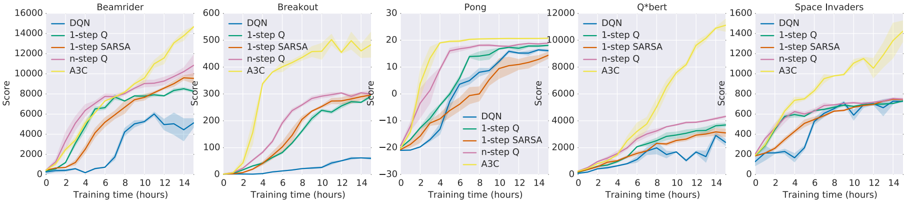
$$\hat{R}_t = r_t + \gamma r_{t+1} + \cdots + \gamma^{T-t+1} r_{T-1} + \gamma^{T-t} V(s_t)$$
$$\hat{A}_t = \hat{R}_t - V(s_t)$$

  $\hat{R}_t$ is target value function, in regression problem
  $\hat{A}_t$ is estimated advantage function
  Compute loss gradient $g = \nabla_\theta \sum_{t=1}^{T} \left[-\log \pi_\theta(a_t \mid s_t) \hat{A}_t + c(V(s) - \hat{R}_t)^2 \right]$
  g is plugged into a stochastic gradient descent variant, e.g., Adam.
end for

V. Mnih, A. P. Badia, M. Mirza, et al. "Asynchronous Methods for Deep Reinforcement Learning". In: *ICML* (2016).

A3C Video

A3C Results


Beamrider
Breakout
Pong
Q*bert
Space Invaders
DQN
1-step Q
1-step SARSA
n-step Q
A3C
Score
Training time (hours)

TD(λ) Methods: Generalized Advantage Estimation

- Recall, finite-horizon advantage estimators

$$\hat{A}_t^{(k)} = r_t + \gamma r_{t+1} + \cdots + \gamma^{k-1} r_{t+k-1} + \gamma^k V(s_{t+k}) - V(s_t)$$

- Define the TD error $\delta_t = r_t + \gamma V(s_{t+1}) - V(s_t)$
- By a telescoping sum,

$$\hat{A}_t^{(k)} = \delta_t + \gamma \delta_{t+1} + \cdots + \gamma^{k-1} \delta_{t+k-1}$$

- Take exponentially weighted average of finite-horizon estimators:

$$\hat{A}^{\lambda} = \hat{A}_t^{(1)} + \lambda \hat{A}_t^{(2)} + \lambda^2 \hat{A}_t^{(3)} + \dots$$

- We obtain

$$\hat{A}_t^{\lambda} = \delta_t + (\gamma\lambda)\delta_{t+1} + (\gamma\lambda)^2 \delta_{t+2} + \dots$$

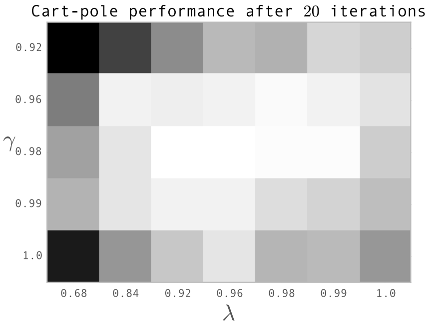
- This scheme named *generalized advantage estimation* (GAE) in [1], though versions have appeared earlier, e.g., [2]. Related to TD(λ)

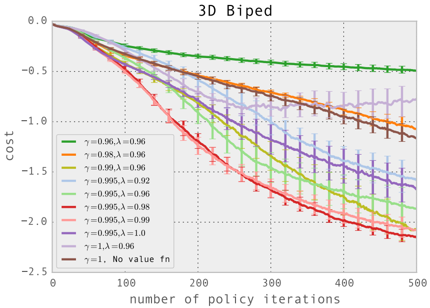
J. Schulman, P. Moritz, S. Levine, et al. "High-dimensional continuous control using generalized advantage estimation". In: *ICML*. 2015

H. Kimura and S. Kobayashi. "An Analysis of Actor/Critic Algorithms Using Eligibility Traces: Reinforcement Learning with Imperfect Value Function." In: *ICML*. 1998, pp. 278–286

Choosing parameters γ, λ

Performance as γ, λ are varied



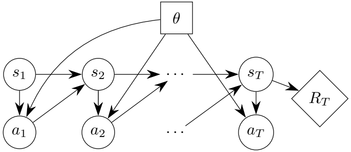


TRPO+GAE Video

Pathwise Derivative Policy Gradient Methods

Deriving the Policy Gradient, Reparameterized

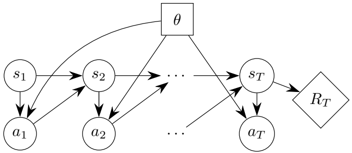
- Episodic MDP:



Want to compute $\nabla_\theta \mathbb{E}\left[R_T\right]$. We'll use $\nabla_\theta \log \pi(a_t \mid s_t; \theta)$

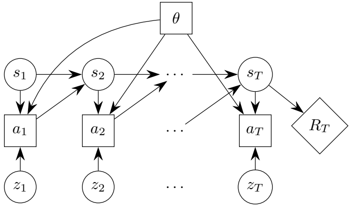
Deriving the Policy Gradient, Reparameterized

- Episodic MDP:



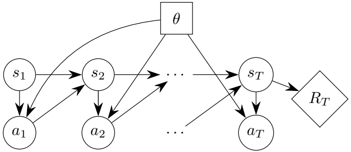
Want to compute $\nabla_\theta \mathbb{E}[R_T]$. We'll use $\nabla_\theta \log \pi(a_t \mid s_t; \theta)$

- Reparameterize: $a_t = \pi(s_t, z_t; \theta)$. z_t is noise from fixed distribution.



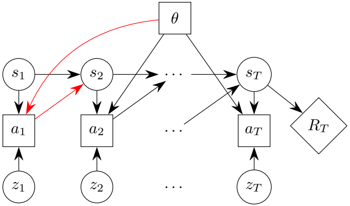
Deriving the Policy Gradient, Reparameterized

- Episodic MDP:



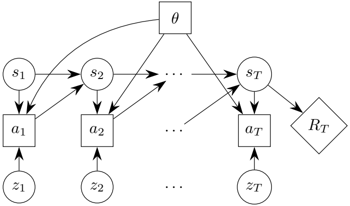
Want to compute $\nabla_\theta \mathbb{E}[R_T]$. We'll use $\nabla_\theta \log \pi(a_t \mid s_t; \theta)$

- Reparameterize: $a_t = \pi(s_t, z_t; \theta)$. z_t is noise from fixed distribution.



- Only works if $P(s_2 \mid s_1, a_1)$ is known $\ddot\frown$

Using a Q-function



$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\theta}\mathbb{E}\left[R_T\right] &= \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}R_T}{\mathrm{d}a_t}\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right] = \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}}{\mathrm{d}a_t}\mathbb{E}\left[R_T \mid a_t\right]\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right] \\
&= \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}Q(s_t, a_t)}{\mathrm{d}a_t}\frac{\mathrm{d}a_t}{\mathrm{d}\theta}\right] = \mathbb{E}\left[\sum_{t=1}^{T}\frac{\mathrm{d}}{\mathrm{d}\theta}Q(s_t, \pi(s_t, z_t; \theta))\right]
\end{aligned}
$$

SVG(0) Algorithm

- Learn Q_ϕ to approximate $Q^{\pi,\gamma}$, and use it to compute gradient estimates.

N. Heess, G. Wayne, D. Silver, et al. "Learning continuous control policies by stochastic value gradients". In: *NIPS*. 2015

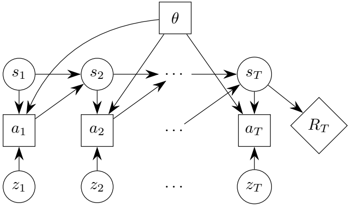
SVG(0) Algorithm

- Learn Q_ϕ to approximate $Q^{\pi,\gamma}$, and use it to compute gradient estimates.
- Pseudocode:

for iteration$=1, 2, \ldots$ **do**
  Execute policy π_θ to collect T timesteps of data
  Update π_θ using $g \propto \nabla_\theta \sum_{t=1}^{T} Q(s_t, \pi(s_t, z_t; \theta))$
  Update Q_ϕ using $g \propto \nabla_\phi \sum_{t=1}^{T} (Q_\phi(s_t, a_t) - \hat{Q}_t)^2$, e.g. with TD($\lambda$)
end for

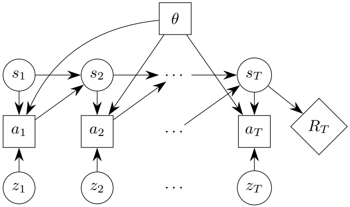
N. Heess, G. Wayne, D. Silver, et al. "Learning continuous control policies by stochastic value gradients". In: *NIPS*. 2015

SVG(1) Algorithm



- Instead of learning Q, we learn
  - State-value function $V \approx V^{\pi,\gamma}$
  - Dynamics model f, approximating $s_{t+1} = f(s_t, a_t) + \zeta_t$
- Given transition (s_t, a_t, s_{t+1}), infer $\zeta_t = s_{t+1} - f(s_t, a_t)$
- $Q(s_t, a_t) = \mathbb{E}\left[r_t + \gamma V(s_{t+1})\right] = \mathbb{E}\left[r_t + \gamma V(f(s_t, a_t) + \zeta_t)\right]$, and $a_t = \pi(s_t, \theta, \zeta_t)$

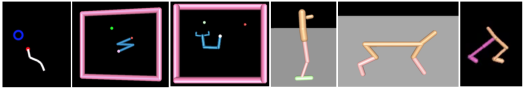
SVG(∞) Algorithm



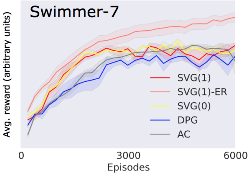
- Just learn dynamics model f
- Given whole trajectory, infer all noise variables
- Freeze all policy and dynamics noise, differentiate through entire deterministic computation graph

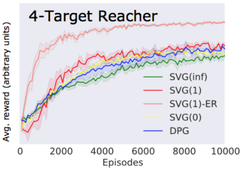
SVG Results

- Applied to 2D robotics tasks



- Overall: different gradient estimators behave similarly





N. Heess, G. Wayne, D. Silver, et al. "Learning continuous control policies by stochastic value gradients". In: *NIPS*. 2015

Deterministic Policy Gradient

- For Gaussian actions, variance of score function policy gradient estimator goes to infinity as variance goes to zero
- But SVG(0) gradient is fine when $\sigma \to 0$

$$\nabla_\theta \sum_t Q(s_t, \pi(s_t, \theta, \zeta_t))$$

- Problem: there's no exploration.
- Solution: add noise to the policy, but estimate Q with TD(0), so it's valid off-policy
- Policy gradient is a little biased (even with $Q = Q^\pi$), but only because state distribution is off—it gets the right gradient at every state

D. Silver, G. Lever, N. Heess, et al. "Deterministic policy gradient algorithms". In: *ICML*. 2014

Deep Deterministic Policy Gradient

- Incorporate replay buffer and target network ideas from DQN for increased stability

T. P. Lillicrap, J. J. Hunt, A. Pritzel, et al. "Continuous control with deep reinforcement learning". In: *ICLR* (2015).

Deep Deterministic Policy Gradient

- Incorporate replay buffer and target network ideas from DQN for increased stability
- Use lagged (Polyak-averaging) version of Q_ϕ and π_θ for fitting Q_ϕ (towards $Q^{\pi,\gamma}$) with TD(0)

$$\hat{Q}_t = r_t + \gamma Q_{\phi'}(s_{t+1}, \pi(s_{t+1}; \theta'))$$

T. P. Lillicrap, J. J. Hunt, A. Pritzel, et al. "Continuous control with deep reinforcement learning". In: *ICLR* (2015)

Deep Deterministic Policy Gradient

- Incorporate replay buffer and target network ideas from DQN for increased stability
- Use lagged (Polyak-averaging) version of Q_ϕ and π_θ for fitting Q_ϕ (towards $Q^{\pi,\gamma}$) with TD(0)

$$\hat{Q}_t = r_t + \gamma Q_{\phi'}(s_{t+1}, \pi(s_{t+1}; \theta'))$$

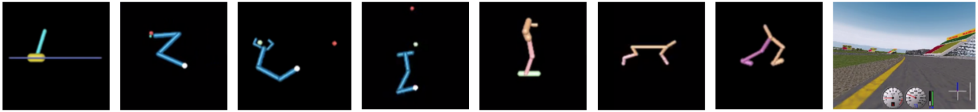
- Pseudocode:

for iteration$=1, 2, \ldots$ **do**
    Act for several timesteps, add data to replay buffer
    Sample minibatch
    Update π_θ using $g \propto \nabla_\theta \sum_{t=1}^{T} Q(s_t, \pi(s_t, z_t; \theta))$
    Update Q_ϕ using $g \propto \nabla_\phi \sum_{t=1}^{T} (Q_\phi(s_t, a_t) - \hat{Q}_t)^2$,
end for

T. P. Lillicrap, J. J. Hunt, A. Pritzel, et al. "Continuous control with deep reinforcement learning". In: *ICLR* (2015)

DDPG Results

Applied to 2D and 3D robotics tasks and driving with pixel input



T. P. Lillicrap, J. J. Hunt, A. Pritzel, et al. "Continuous control with deep reinforcement learning". In: *ICLR* (2015).

Policy Gradient Methods: Comparison

- Two kinds of policy gradient estimator
  - REINFORCE / score function estimator: $\nabla \log \pi(a \mid s)\hat{A}$.
    - Learn Q or V for variance reduction, to estimate $\hat{A}$
  - Pathwise derivative estimators (differentiate wrt action)
    - SVG(0) / DPG: $\frac{\mathrm{d}}{\mathrm{d}a} Q(s, a)$ (learn Q)
    - SVG(1): $\frac{\mathrm{d}}{\mathrm{d}a}(r + \gamma V(s'))$ (learn f, V)
    - SVG(∞): $\frac{\mathrm{d}}{\mathrm{d}a_t}(r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \dots)$ (learn f)
- Pathwise derivative methods more sample-efficient when they work (maybe), but work less generally due to high bias

Policy Gradient Methods: Comparison

Task	Random	REINFORCE	TNPG	RWR	REPS	TRPO	CEM	CMA-ES	DDPG
Cart-Pole Balancing	77.1 ± 0.0	4693.7 ± 14.0	**3986.4 ± 748.9**	**4861.5 ± 12.3**	565.6 ± 137.6	**4869.8 ± 37.6**	4815.4 ± 4.8	2440.4 ± 568.3	4634.4 ± 87.8
Inverted Pendulum*	−153.4 ± 0.2	13.4 ± 18.0	**209.7 ± 55.5**	84.7 ± 13.8	−113.3 ± 4.6	**247.2 ± 76.1**	38.2 ± 25.7	−40.1 ± 5.7	40.0 ± 244.6
Mountain Car	−415.4 ± 0.0	−67.1 ± 1.0	**-66.5 ± 4.5**	−79.4 ± 1.1	−275.6 ± 166.3	**-61.7 ± 0.9**	−66.0 ± 2.4	−85.0 ± 7.7	−288.4 ± 170.3
Acrobot	−1904.5 ± 1.0	−508.1 ± 91.0	−395.8 ± 121.2	−352.7 ± 35.9	−1001.5 ± 10.8	−326.0 ± 24.4	−436.8 ± 14.7	−785.6 ± 13.1	**-223.6 ± 5.8**
Double Inverted Pendulum*	149.7 ± 0.1	4116.5 ± 65.2	**4455.4 ± 37.6**	3614.8 ± 368.1	446.7 ± 114.8	**4412.4 ± 50.4**	2566.2 ± 178.9	1576.1 ± 51.3	2863.4 ± 154.0
Swimmer*	−1.7 ± 0.1	92.3 ± 0.1	**96.0 ± 0.2**	60.7 ± 5.5	3.8 ± 3.3	**96.0 ± 0.2**	68.8 ± 2.4	64.9 ± 1.4	85.8 ± 1.8
Hopper	8.4 ± 0.0	714.0 ± 29.3	**1155.1 ± 57.9**	553.2 ± 71.0	86.7 ± 17.6	**1183.3 ± 150.0**	63.1 ± 7.8	20.3 ± 14.3	267.1 ± 43.5
2D Walker	−1.7 ± 0.0	506.5 ± 78.8	**1382.6 ± 108.2**	136.0 ± 15.9	−37.0 ± 38.1	**1353.8 ± 85.0**	84.5 ± 19.2	77.1 ± 24.3	318.4 ± 181.6
Half-Cheetah	−90.8 ± 0.3	1183.1 ± 69.2	**1729.5 ± 184.6**	376.1 ± 28.2	34.5 ± 38.0	**1914.0 ± 120.1**	330.4 ± 274.8	441.3 ± 107.6	**2148.6 ± 702.7**
Ant*	13.4 ± 0.7	548.3 ± 55.5	**706.0 ± 127.7**	37.6 ± 3.1	39.0 ± 9.8	**730.2 ± 61.3**	49.2 ± 5.9	17.8 ± 15.5	326.2 ± 20.8
Simple Humanoid	41.5 ± 0.2	128.1 ± 34.0	**255.0 ± 24.5**	93.3 ± 17.4	28.3 ± 4.7	**269.7 ± 40.3**	60.6 ± 12.9	28.7 ± 3.9	99.4 ± 28.1
Full Humanoid	13.2 ± 0.1	262.2 ± 10.5	**288.4 ± 25.2**	46.7 ± 5.6	41.7 ± 6.1	**287.0 ± 23.4**	36.9 ± 2.9	N/A ± N/A	119.0 ± 31.2
Cart-Pole Balancing (LS)*	77.1 ± 0.0	420.9 ± 265.5	**945.1 ± 27.8**	68.9 ± 1.5	898.1 ± 22.1	**960.2 ± 46.0**	227.0 ± 223.0	68.0 ± 1.6	
Inverted Pendulum (LS)	−122.1 ± 0.1	−13.4 ± 3.2	**0.7 ± 6.1**	−107.4 ± 0.2	−87.2 ± 8.0	**4.5 ± 4.1**	−81.2 ± 33.2	−62.4 ± 3.4	
Mountain Car (LS)	−83.0 ± 0.0	−81.2 ± 0.6	**-65.7** ± 9.0	−81.7 ± 0.1	−82.6 ± 0.4	**-64.2 ± 9.5**	**-68.9 ± 1.3**	**-73.2 ± 0.6**	
Acrobot (LS)*	−393.2 ± 0.0	−128.9 ± 11.6	**-84.6 ± 2.9**	−235.9 ± 5.3	−379.5 ± 1.4	**-83.3 ± 9.9**	−149.5 ± 15.3	−159.9 ± 7.5	
Cart-Pole Balancing (NO)*	101.4 ± 0.1	616.0 ± 210.8	**916.3 ± 23.0**	93.8 ± 1.2	99.6 ± 7.2	606.2 ± 122.2	181.4 ± 32.1	104.4 ± 16.0	
Inverted Pendulum (NO)	−122.2 ± 0.1	6.5 ± 1.1	**11.5 ± 0.5**	−110.0 ± 1.4	−119.3 ± 4.2	**10.4 ± 2.2**	−55.6 ± 16.7	−80.3 ± 2.8	
Mountain Car (NO)	−83.0 ± 0.0	−74.7 ± 7.8	**-64.5 ± 8.6**	−81.7 ± 0.1	−82.9 ± 0.1	**-60.2 ± 2.0**	−67.4 ± 1.4	−73.5 ± 0.5	
Acrobot (NO)*	−393.5 ± 0.0	**-186.7 ± 31.3**	**-164.5 ± 13.4**	−233.1 ± 0.4	−258.5 ± 14.0	**-149.6 ± 8.6**	−213.4 ± 6.3	−236.6 ± 6.2	
Cart-Pole Balancing (SI)*	76.3 ± 0.1	431.7 ± 274.1	**980.5 ± 7.3**	69.0 ± 2.8	702.4 ± 196.4	**980.3 ± 5.1**	746.6 ± 93.2	71.6 ± 2.9	
Inverted Pendulum (SI)	−121.8 ± 0.2	−5.3 ± 5.6	**14.8 ± 1.7**	−108.7 ± 4.7	−92.8 ± 23.9	**14.1 ± 0.9**	−51.8 ± 10.6	−63.1 ± 4.8	
Mountain Car (SI)	−82.7 ± 0.0	−63.9 ± 0.2	**-61.8 ± 0.4**	−81.4 ± 0.1	−80.7 ± 2.3	**-61.6 ± 0.4**	−63.9 ± 1.0	−66.9 ± 0.6	
Acrobot (SI)*	−387.8 ± 1.0	**-169.1 ± 32.3**	**-156.6 ± 38.9**	−233.2 ± 2.6	−216.1 ± 7.7	**-170.9 ± 40.3**	−250.2 ± 13.7	−245.0 ± 5.5	
Swimmer + Gathering	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0
Ant + Gathering	−5.8 ± 5.0	−0.1 ± 0.1	−0.4 ± 0.1	−5.5 ± 0.5	−6.7 ± 0.7	−0.4 ± 0.0	−4.7 ± 0.7	N/A ± N/A	−0.3 ± 0.3
Swimmer + Maze	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0
Ant + Maze	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	0.0 ± 0.0	N/A ± N/A	0.0 ± 0.0

Y. Duan, X. Chen, R. Houthooft, et al. "Benchmarking Deep Reinforcement Learning for Continuous Control". In: *ICML* (2016)

Stochastic Computation Graphs

Gradients of Expectations

Want to compute $\nabla_\theta \mathbb{E}[F]$. Where's θ?

- In distribution, e.g., $\mathbb{E}_{x \sim p(\cdot \mid \theta)}[F(x)]$
  - $\nabla_\theta \mathbb{E}_x[f(x)] = \mathbb{E}_x[f(x) \nabla_\theta \log p_x(x;\ \theta)]$.
  - Score function estimator
  - Example: REINFORCE policy gradients, where x is the trajectory
- Outside distribution: $\mathbb{E}_{z \sim \mathcal{N}(0,1)}[F(\theta, z)]$

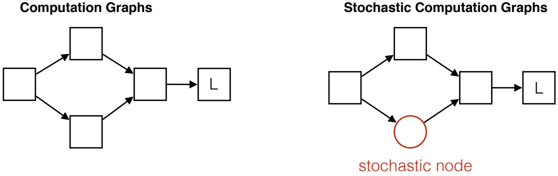
$$\nabla_\theta \mathbb{E}_z[f(x(z,\theta))] = \mathbb{E}_z[\nabla_\theta f(x(z,\theta))].$$

  - Pathwise derivative estimator
  - Example: SVG policy gradient
- Often, we can reparametrize, to change from one form to another
- What if F depends on θ in complicated way, affecting distribution and F?

M. C. Fu. "Gradient estimation". In: *Handbooks in operations research and management science* 13 (2006), pp. 575–616

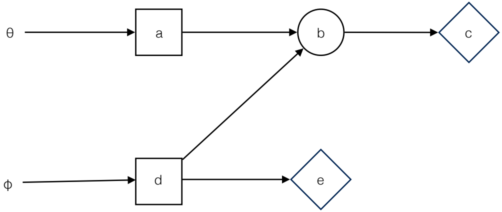
Stochastic Computation Graphs

- Stochastic computation graph is a DAG, each node corresponds to a deterministic or stochastic operation
- Can automatically derive unbiased gradient estimators, with variance reduction



J. Schulman, N. Heess, T. Weber, et al. "Gradient Estimation Using Stochastic Computation Graphs". In: *NIPS*. 2015

Worked Example



- $L = c + e$. Want to compute $\frac{\mathrm{d}}{\mathrm{d}\theta}\mathbb{E}[L]$ and $\frac{\mathrm{d}}{\mathrm{d}\phi}\mathbb{E}[L]$.
- Treat stochastic nodes (b, d) as constants, and introduce losses $logprob * (futurecost)$ at each stochastic node
- Obtain unbiased gradient estimate by differentiating surrogate:

$$\mathrm{Surrogate}(\theta, \psi) = \underbrace{c + e}_{(1)} + \underbrace{\log p(\hat{b} \mid a, d)\hat{c}}_{(2)}$$

(1): how parameters influence cost through deterministic dependencies

(2): how parameters affect distribution over random variables.

Outline

- Derivative free methods
  - Cross Entropy Method (CEM) / Finite Differences / Fixing Random Seed
- Likelihood Ratio (LR) Policy Gradient
  - Derivation / Connection w/Importance Sampling
- Natural Gradient / Trust Regions (-> TRPO)
- Variance Reduction using Value Functions (Actor-Critic) (-> GAE, A3C)
- Pathwise Derivatives (PD) (-> DPG, DDPG, SVG)
- Stochastic Computation Graphs (generalizes LR / PD)
- ***Guided Policy Search (GPS)***
- Inverse Reinforcement Learning

Goal

- Find parameterized policy $\pi_\theta(\mathbf{u}_t|\mathbf{x}_t)$ that optimizes:

$$J(\theta) = \sum_{t=1}^{T} \mathrm{E}_{\pi_\theta(\mathbf{x}_t,\mathbf{u}_t)}[l(\mathbf{x}_t, \mathbf{u}_t)]$$

- Notation: $$\pi_\theta(\tau) = p(\mathbf{x}_1) \prod_{t=1}^{T} p(\mathbf{x}_{t+1}|\mathbf{x}_t, \mathbf{u}_t)\pi_\theta(\mathbf{u}_t|\mathbf{x}_t)$$

$$\tau = \{\mathbf{x}_1, \mathbf{u}_1, \ldots, \mathbf{x}_T, \mathbf{u}_T\}$$

- RL takes lots of data… Can we reduce to supervised learning?

Naïve Solution

- Step 1:
  - Consider sampled problem instances $i = 1, 2, \ldots, I$
  - Find a trajectory-centric controller $\pi_i(\mathbf{u}_t|\mathbf{x}_t)$ for each problem instance
- Step 2:
  - Supervised training of neural net to match all $\pi_i(\mathbf{u}_t|\mathbf{x}_t)$

$$\pi_\theta \leftarrow \arg\min_\theta \sum_i D_{\mathrm{KL}}(p_i(\tau)||\pi_\theta(\tau))$$

- ISSUES:
  - Compounding error (Ross, Gordon, Bagnell JMLR 2011 "Dagger")
  - Mismatch train vs. test E.g., Blind peg, Vision,…

(Generic) Guided Policy Search

- Optimization formulation:

$$\min_{\theta, p_1, \ldots, p_N} \sum_{i=1}^{N} \sum_{t=1}^{T} E_{p_i(\mathbf{x}_t, \mathbf{u}_t)}[\ell(\mathbf{x}_t, \mathbf{u}_t)] \text{ such that } p_i(\mathbf{u}_t|\mathbf{x}_t) = \pi_\theta(\mathbf{u}_t|\mathbf{x}_t) \;\; \forall \mathbf{x}_t, \mathbf{u}_t, t, i. \qquad (1)$$

Particular form of the constraint varies depending on the specific method:

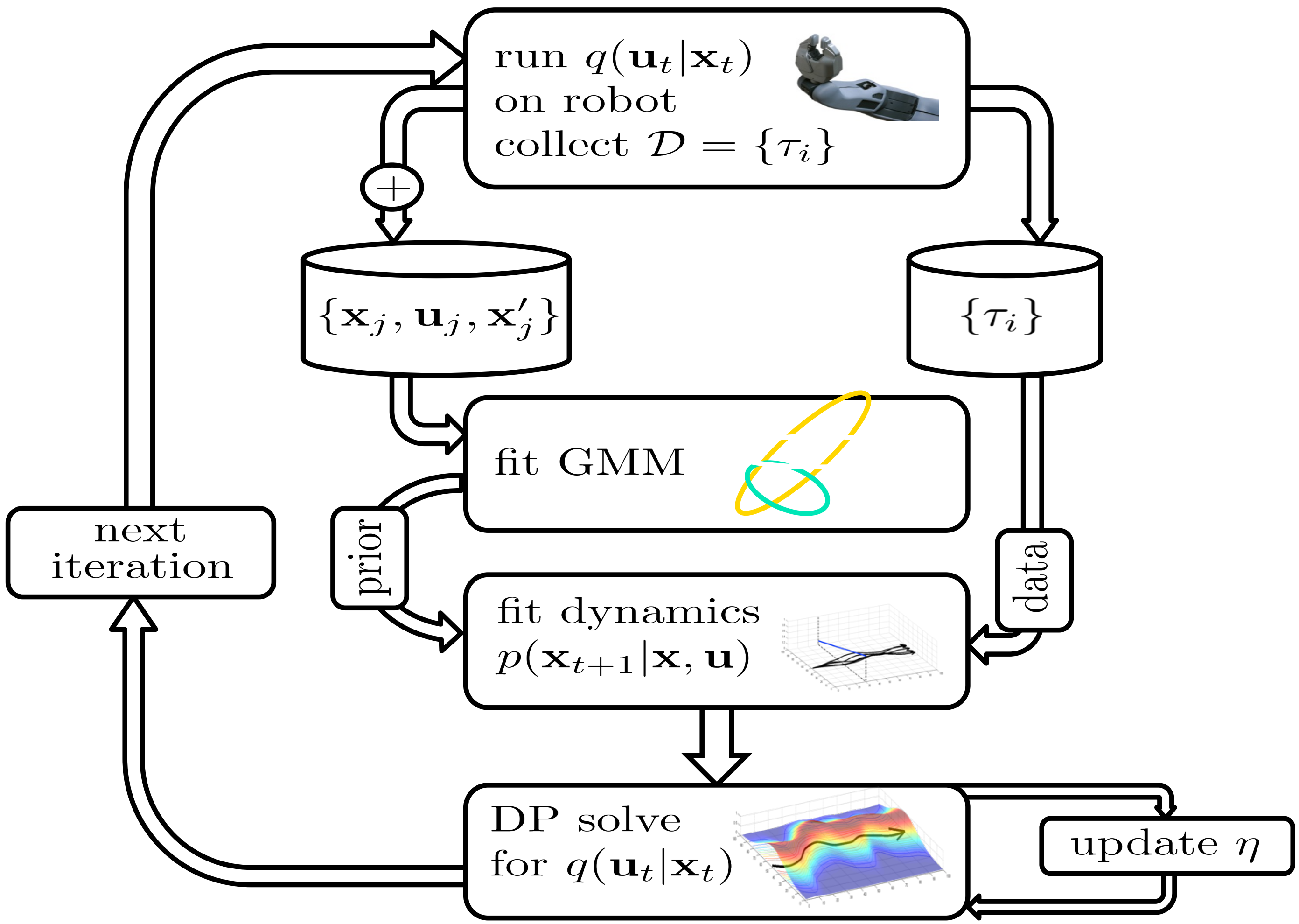
Dual gradient descent: Levine and Abbeel, NIPS 2014

Penalty methods: Mordatch, Lowrey, Andrew, Popovic, Todorov, NIPS 2016

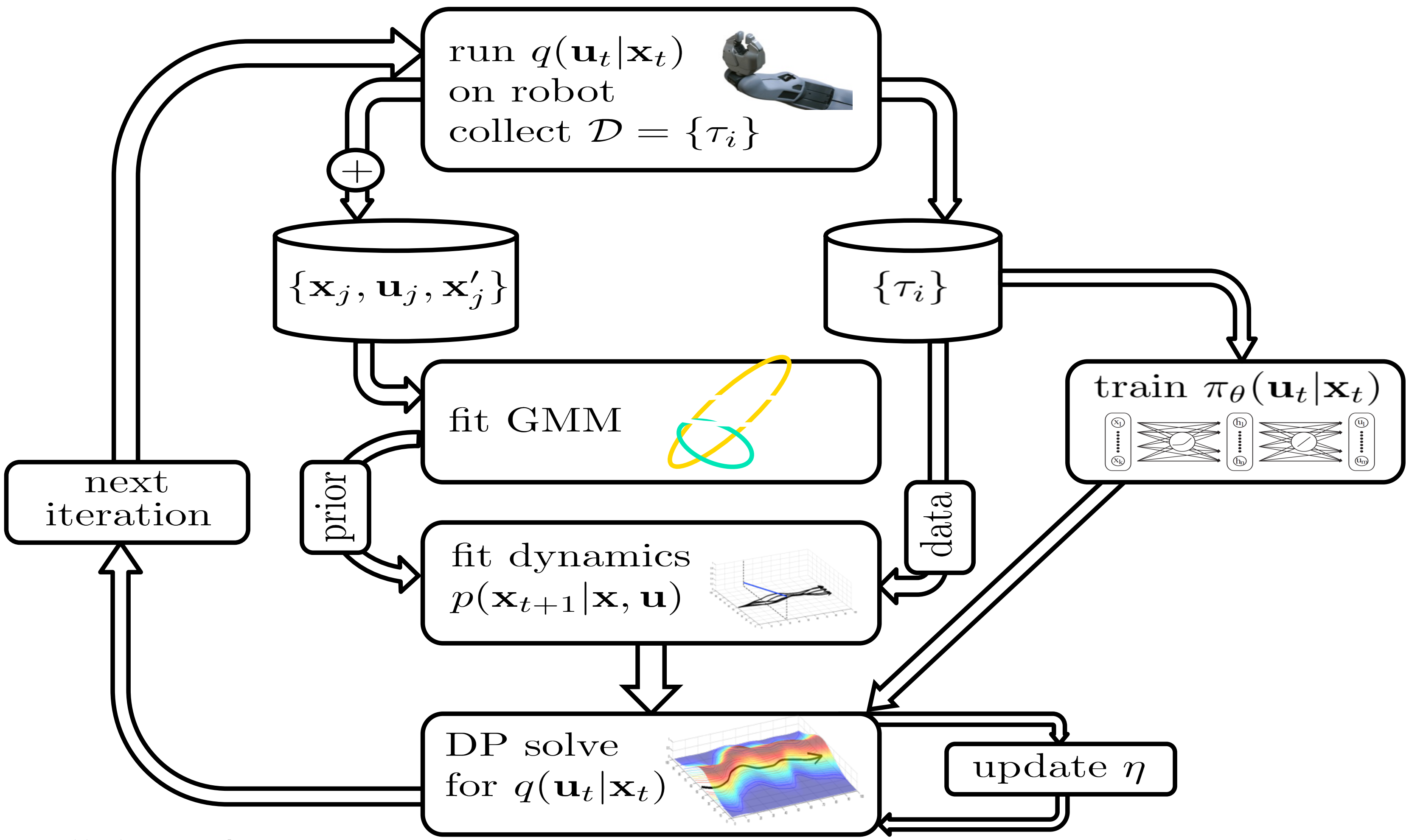
ADMM: Mordatch and Todorov, RSS 2014

Bregman ADMM: Levine, Finn, Darrell, Abbeel, JMLR 2016

Mirror Descent: Montgomery, Levine, NIPS 2016

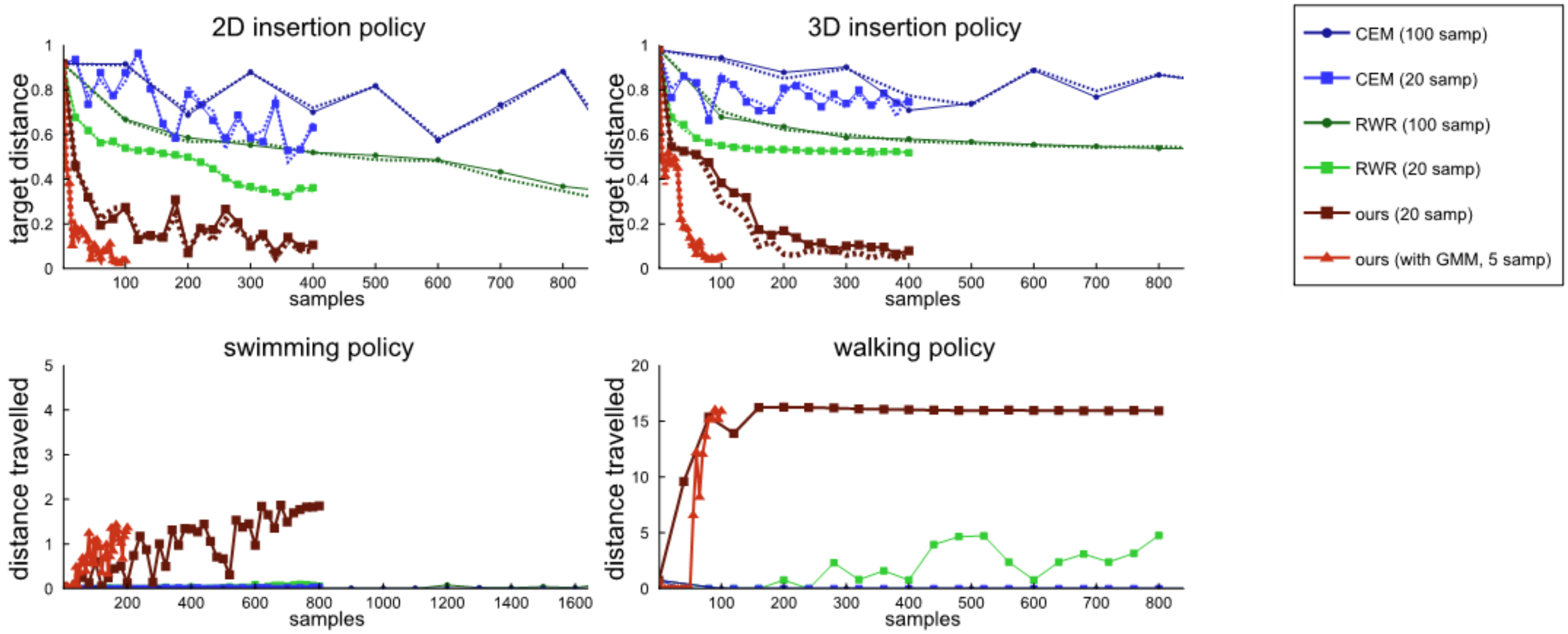


[Levine & Abbeel, NIPS 2014]



[Levine & Abbeel, NIPS 2014]

Comparison

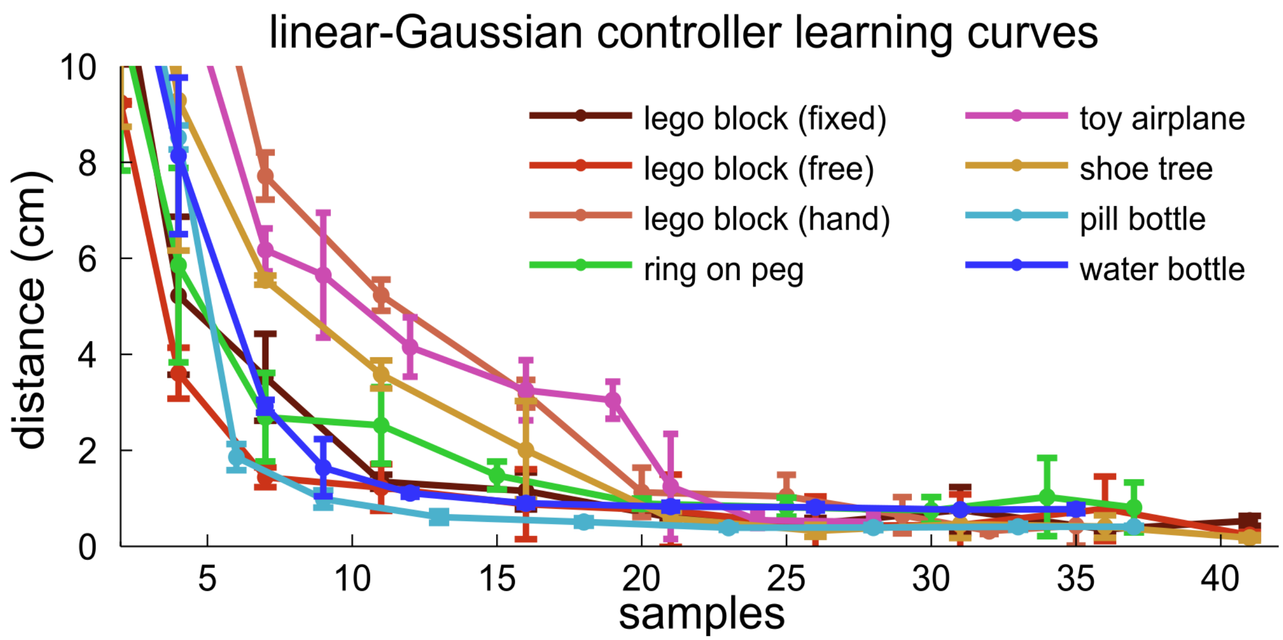


[Levine, Wagener, Abbeel, ICRA 2015]

Block Stacking – Learning the Controller for a Single Instance

[Levine, Wagener, Abbeel, ICRA 2015]

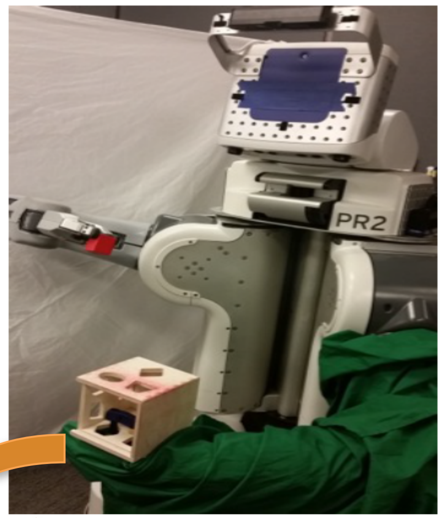
Linear-Gaussian Controller Learning Curves



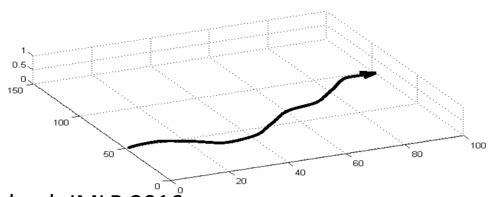
[Levine, Wagener, Abbeel, ICRA 2015]

Instrumented Training

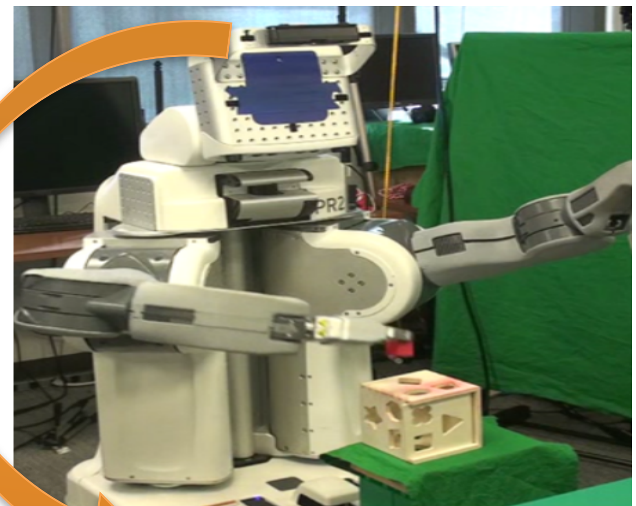
training time



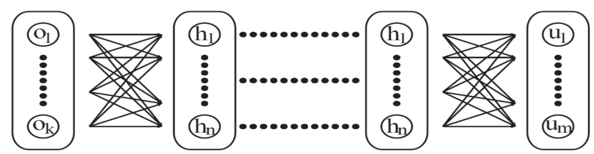
$\mathbf{x}_t \longrightarrow \mathbf{u}_t$



test time

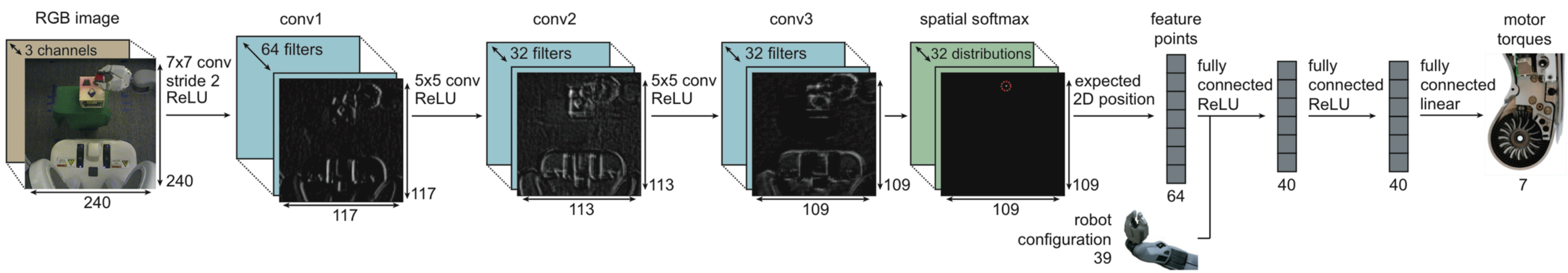


$\mathbf{o}_t \longrightarrow \mathbf{u}_t$



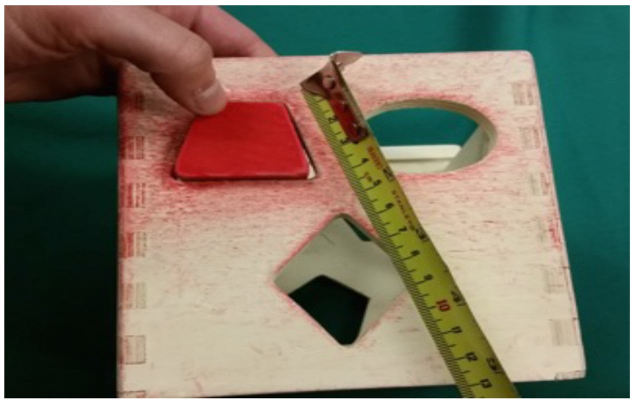
[Levine*, Finn*, Darrell, Abbeel, JMLR 2016]

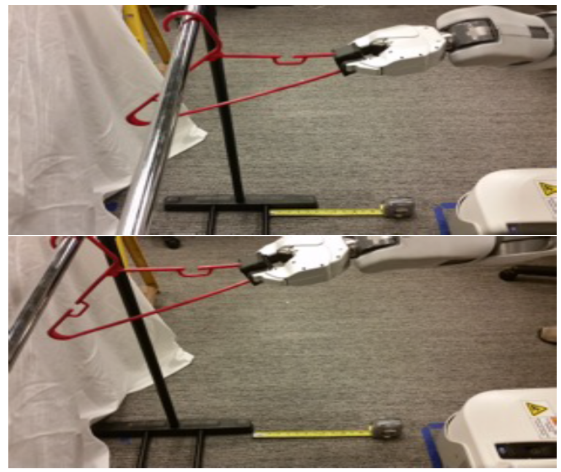
Architecture (92,000 parameters)



[Levine*, Finn*, Darrell, Abbeel, JMLR 2016

Experimental Tasks





[Levine*, Finn*, Darrell, Abbeel, JMLR 2016

Learning

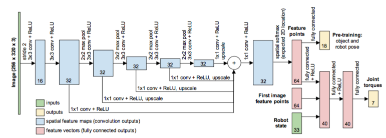
[Levine*, Finn*, Darrell, Abbeel, JMLR 2016

Learned Skills

[Levine*, Finn*, Darrell, Abbeel, JMLR 2016

PI-GPS

- Uses PI2 (rather than iLQG) as the trajectory optimizer
  - In these experiments:
    - PI2 optimizes over sequence of linear feedback controllers
    - PI2 initialized from demonstrations
    - Neural net architecture:



[Chebotar, Kalakrishnan, Yahya, Li, Schaal, Levine, arXiv 2016]

Outline

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- Guided Policy Search (GPS)
- ***Current Frontiers***

Current Frontiers (+pointers to some representative recent work)

- Off-policy Policy Gradients / Off-policy Actor Critic / Connect with Q-Learning
  - DDPG [Lillicrap et al, 2015]; Q-prop [Gu et al, 2016]; Doubly Robust [Dudik et al, 2011], …
  - PGQ [O'Donoghue et al, 2016]; ACER [Wang et al, 2016]; Q(lambda) [Harutyunyan et al, 2016]; Retrace(lambda) [Munos et al, 2016]…
- Exploration
  - VIME [Houthooft et al, 2016]; Count-Based Exploration [Bellemare et al, 2016]; #Exploration [Tang et al, 2016]; Curiosity [Schmidhueber, 1991]; …
- Auxiliary objectives
  - Learning to Navigate [Mirowski et al, 2016]; RL with Unsupervised Auxiliary Tasks [Jaderberg et al, 2016], …
- Multi-task and transfer (incl. sim2real)
  - DeepDriving [Chen et al, 2015]; Progressive Nets [Rusu et al, 2016]; Flight without a Real Image [Sadeghi & Levine, 2016]; Sim2Real Visuomotor [Tzeng et al, 2016]; Sim2Real Inverse Dynamics [Christiano et al, 2016]; Modular NNs [Devin*, Gupta*, et al 2016]
- Language
  - Learning to Communicate [Foerster et al, 2016]; Multitask RL w/Policy Sketches [Andreas et al, 2016]; Learning Language through Interaction [Wang et al, 2016]

Current Frontiers (+pointers to some representative recent work)

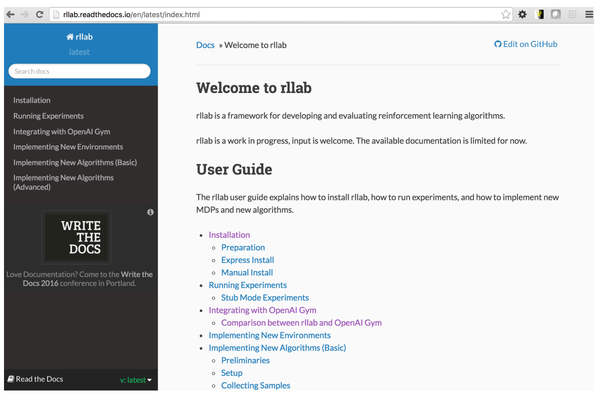
- Meta-RL
  - RL2: Fast RL through Slow RL [Duan et al., 2016]; Learning to Reinforcement Learn [Wang et al, 2016]; Learning to Experiment [Denil et al, 2016]; Learning to Learn for Black-Box Opt. [Chen et al, 2016], …
- 24/7 Data Collection
  - Learning to Grasp from 50K Tries [Pinto&Gupta, 2015]; Learning Hand-Eye Coordination [Levine et al, 2016]; Learning to Poke by Poking [Agrawal et al, 2016]
- Safety
  - Survey: Garcia and Fernandez, JMLR 2015
- Architectures
  - Memory, Active Perception in Minecraft [Oh et al, 2016]; DRQN [Hausknecht&Stone, 2015]; Dueling Networks [Wang et al, 2016]; …
- Inverse RL
  - Generative Adversarial Imitation Learning [Ho et al, 2016]; Guided Cost Learning [Finn et al, 2016]; MaxEnt Deep RL [Wulfmeier et al, 2016]; …
- Model-based RL
  - Deep Visual Foresight [Finn & Levine, 2016]; Embed to Control [Watter et al., 2015]; Spatial Autoencoders Visuomotor Learning [Finn et al, 2015]; PILCO [Deisenroth et al, 2015]
- Hierarchical RL
  - Modulated Locomotor Controllers [Heess et al, 2016]; STRAW [Vezhnevets et al, 2016]; Option-Critic [Bacon et al, 2016]; h-DQN [Kulkarni et al, 2016]; Hierarchical Lifelong Learning in Minecraft [Tessler et al, 2016]

How to Learn More and Get Started?

- **(1) Deep RL Courses**
  - CS294-112 Deep Reinforcement Learning (UC Berkeley): http://rll.berkeley.edu/deeprlcourse/ by Sergey Levine, John Schulman, Chelsea Finn
  - COMPM050/COMPGI13 Reinforcement Learning (UCL): http://www0.cs.ucl.ac.uk/staff/D.Silver/web/Teaching.html by David Silver

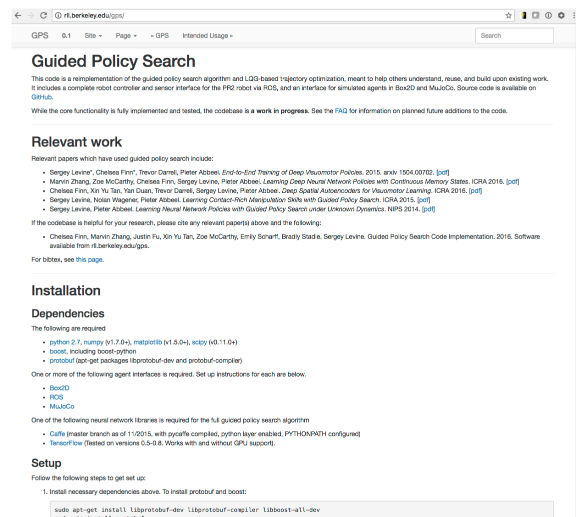
How to Learn More and Get Started?

- **(2) Deep RL Code Bases**
  - **rllab:** https://github.com/openai/rllab
    Duan, Chen, Houthooft, Schulman et al



  - Rlpy: https://rlpy.readthedocs.io/en/latest/
    Geramifard, Klein, Dann, Dabney, How

  - **GPS:** http://rll.berkeley.edu/gps/
    Finn, Zhang, Fu, Tan, McCarthy, Scharff, Stadie, Levine

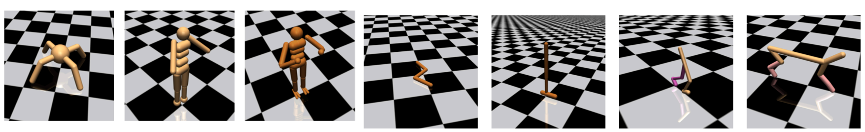


How to Learn More and Get Started?

- **(3) Environments**
  - **Arcade Learning Environment (ALE)** (Bellemare et al, JAIR 2013)

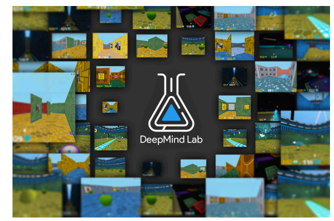
...

  - **MuJoCo:** http://mujoco.org (Todorov)

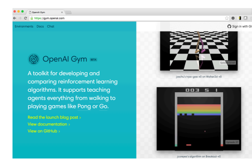


  - **Minecraft** (Microsoft)

- **Deepmind Lab / Labyrinth** (Deepmind)



- **OpenAI Gym:** https://gym.openai.com/



- **Universe:** https://universe.openai.com/

Universe

A software platform for measuring and training an AI's general intelligence across the world's supply of games, websites and other applications.

https://universe.openai.com

Universe -- Games

Release consists of a thousand environments including Flash games, browser tasks, and games like slither.io, StarCraft and GTA V.

https://universe.openai.com

Universe – World of Bits (WoB): “Mini-WoB”

AI follows instructions

https://universe.openai.com

Universe – World of Bits: Real Browser Tasks

AI books plane tickets

https://universe.openai.com

Universe – World of Bits: Educational Games

AI goes to school ☺

https://universe.openai.com

Universe

- Opportunities:
  - Train agents on Universe tasks.
  - Grant us permission to use your game, program, website, or app
  - Integrate new environments.
  - Contribute demonstrations.

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Summary

- Derivative free methods
  - Cross Entropy Method (CEM) / Finite Differences / Fixing Random Seed
- Likelihood Ratio (LR) Policy Gradient
  - Derivation / Connection w/Importance Sampling
- Natural Gradient / Trust Regions (-> TRPO)
- Actor-Critic (-> GAE, A3C)
- Path Derivatives (PD) (-> DPG, DDPG, SVG)
- Stochastic Computation Graphs (generalizes LR / PD)
- Guided Policy Search (GPS)
- Current Frontiers