We thank all the reviewers for their detailed and positive reviews on our manuscript. We respond to some of the questions and comments below. A further round of polishing has been conducted to improve the quality of the paper.

1. Motivation and Practical Use Case of the Neighboring Reward Functions

**Motivation.** As the motivation of our work is to protect the reward function, the mathematical objective is then to make two reward function \( r \) and \( r' \) indistinguishable as long as they are ‘close’ to each other. This ‘close’ description should be defined rigorously by some discrepancy measure between functions. The \( \ell_{\infty} \)-norm we used is general and natural. Alternatively, it is also possible to use the distance metric in an RKHS, namely, \( \langle r, r' \rangle / \| r \|_H \| r' \|_H \). But this requires an assumption that \( r \in \mathcal{H} \) for some pre-defined \( \mathcal{H} \). Hence it is less relevant than the \( \ell_{\infty} \)-norm.

**Use case.** The practical use case depends on the exact implementation of the reward function. An example in the recommendation system: if the system records the clickthrough history of the users and the state \( s \) which leads to the clickthrough, then the reward function can be simulated by using kernel density estimation over \( s \) on these clickthroughs. Then, removing one instance of clickthrough incurs a maximum change of a constant to the infinity norm; Another example is when the reward function is the average of the utility functions of \( N \) users. Removing one user will change the infinity norm by at most \( C_1 / N \), as long as these utility functions are bounded by \( C_1 \).

Overall, our notion of privacy and neighborhood is general enough to be applied to a variety of practical problems.

2. Explanation of Algorithm 1

**Adding noise to \( r(\cdot) \).** Adding the noise directly to \( r(\cdot) \) is the input perturbation method to preserve privacy. Namely, if we sample \( g \in \mathcal{G}(0, \sigma^2 K) \) and replace \( r(\cdot) \) in the vanilla deep Q-learning algorithm by \( r(\cdot) + g(\cdot) \), then by Proposition 4 the algorithm is differentially private. However, input perturbation is usually less preferred as it tends to incur a high utility loss. We have illustrated in Figure 2 (blue curve) that it underperforms our algorithm significantly.

**Intuition.** The intuition behind the algorithm is to add functional noise to \( Q(\cdot) \). Line 14-18 are an algorithmic implementation of the Gaussian process (under the Sobolev space and kernel in Lemma 6). More intuitively, we can regard line 14-18 as generating \( g \sim \mathcal{G}(0, \sigma^2 K) \). Then, whenever \( Q(s, \cdot) \) is queried (in line 12, 19, and 20), \( Q(s, \cdot) + g(s) \) is returned instead. We have revised our manuscript and commented this intuition on the side of the algorithm. Therefore the intuition and the discrete implementation will be easier to understand.

**Clarity.** We have made the following revisions for clarity: A. In the term \( C(\alpha, k, L, B) \) in line 5 of the algorithm, \( k \) is a free parameter. It is the tail bound \( u/2 \) in Lemma 8 that balances the noise level \( \sigma \) and the approximation factor \( \delta + J \exp(-2k - 8.68\sqrt{3}\sigma)^2/2 \). For clarity, we have added \( k \) to line 2 of Algorithm 1 and then discussed the intuition \( k = u/2 \) before Lemma 8. B. In line 16 of the algorithm, \( \mu_{at} \) and \( d_{at} \) are defined in Equation (2), which is in the appendix. We moved (2) to above Proposition 9 and modified line 16 to Compute \( \mu_{at} \) and \( d_{at} \) according to Equation (2). Then sample \( z_{at} \sim \mathcal{N}(\mu_{at}, d_{at}) \). C. \( \hat{g}(\mathcal{B}) \) denotes a linked list of tuple \((s, z)\), pre-allocated with size \( B \) of memory. Whenever a new \( s \) is queried, the noise \( z \) is calculated in line 16. Then \( (s, z) \) is inserted to (already sorted) \( \hat{g} \) so that \( \hat{g} \) keeps sorted by \( s \). Finding the position to find is done by binary search, namely, bisect.bisect in our Python implementation. D. We have shortened the proof of Theorem 5 into a proof sketch to save space for the explanations.

3. Utility Analysis in Proposition 10

**Original proposition without \( T \).** The number of iteration rounds \( T \) is not involved in our Proposition 10. The reason is that Q-learning algorithms are proved to converge in the discrete state settings. Hence, we consider only the optimal point that the algorithm will converge to. Denote the optimal points under \( Q(\cdot) \) as \( v^* \). By Theorem 1 of Szepesvári’s book [Sze10], the infinity norm \( \| r(s, a) \|_\infty \) is bounded. Q-learning converges in terms of an exponentially decreasing error \( 2\gamma T r_0/(1 - \gamma) \) with respect to the number of iteration rounds \( T' = T / B \). By the triangle inequality \( \| v' - v^* \|_1 \leq \| v' - v^* \|_1 + \| v' - v^* \|_1 + \| v - v^* \|_1 \leq 2n \cdot 2\gamma T r_0 / (1 - \gamma) \). Therefore, Proposition 10 can be re-written as

\[
\mathbb{E} \left[ \frac{1}{n} \| v' - v^* \|_1 \right] \leq \frac{2\sqrt{2\sigma}}{1 - \gamma} + \frac{4\gamma T' r_0}{1 - \gamma},
\]

where the bound is strictly decreasing with the number of iterations rounds \( T' \). We believe the confusion by Reviewer #3 is due to our omitting of the \( O(\gamma T') \) term. As we have revised and added this term back, it should have been clarified.