Reviewer 1: “The algorithm is rather similar to the denoising algorithm referred to in the paper. The denoising algorithm could be performed in a distributed manner as well since it performs calculations on different subsamples of the data.” The second part of our paper (denoised bigNN) may be viewed as an improvement of the denoising algorithm since it not only leverages the denoising technique to shorten the prediction time, but also shortens the preprocessing time by distributed learning. Indeed, the two algorithms share some similarity. However, a subtle difference lies in the fact that the denoising algorithm performs distributed calculation during the prediction time only, while our denoised bigNN distributes the calculation during both the preprocessing time and the prediction time.

“In the extreme cases... it seems there may be a possibility that k > 1/|S|.” Because 1/|S| is a fraction, it is always true that k > N/s, i.e., k could be greater than the subsample size. We would like to clarify that this would never happen since our choice \( k = k_o n^{2 \alpha/(2 \alpha + 1)} s^{-1/(2 \alpha + 1)} \) automatically satisfies \( k < n = N/s \) for a fixed \( k_o \) and large enough \( N \).

“The idea of distributing the data and thus speed up the process of classification seems somewhat inferior to compressing the data. The speedup occurs from multiprocessing rather than reduction of computations (There is still need to entirely search each subsample). The denoising variation of the algorithm presents actual reduction in computation time, which the original BigNN does not...” It depends on how one defines computation time. If one also takes into account the preprocessing time, denoising (or data compression) merely shifts most of the computation time from the prediction stage to the preprocessing stage: but the computation still has to be done. One could argue that the prediction time is what really matters. However, without the likes of the proposed distributed algorithm, preprocessing will be very difficult due to the very large data volume. Our proposed denoised bigNN algorithm not only shortens the preprocessing time of the original denoising algorithm, but also retains the optimal accuracy. We thank you for the other positive comments and will address the relation between the proposed method and the denoising method more clearly in the camera-ready version, if accepted.

Reviewer 2: “Given the aim of the paper, I think it’s extremely important to experimentally compare against recently proposed quantization schemes that are largely about how to scale up nearest neighbors to large datasets: see KV17, and KWS17.” KWS17 is theory oriented and the authors did not provide code. We tried the MATLAB/C++ based code for the KV17 method on Simulation 3. To reach optimal accuracy, KV17 relies on tuning two parameters knob \( \alpha \) and bandwidth \( h \). We tuned \( h \) within \((0.1, 0.2, \ldots, 10)\) and \( \alpha \) within \((1/6, 2/6, \ldots, 1)\). The best KV17 classifier gave a regret 1.5 times of the denoised bigNN method. The preprocessing time (including tuning) is at the order of 4,000 seconds, compared to hundreds seconds using our method. In terms of prediction time, the best speedup (x8) comes at the cost of the worst accuracy (2 times our regret) while the speedup with their best accuracy is only 2 to 3 folds (compared to 10 folds in our methods). Note our R-based code (to be released publicly) has room for improvement. Philosophically quantization schemes share similarity with the denoising scheme: both referred works start with a \( r \)-net, which is a collection of data points that quantize the training data. They correspond to cells of a Voronoi partition of the entire sample space. The average \( Y \) value or the majority class of those training points that fall into each cell is then assigned to these cells. This is quite similar to the denoising scheme in Xue and Kpotufe (2018), in which quantization is achieved by random subsamplings, and pre-labelling the points in the subsample in the denoising algorithm works like “averaging the \( Y \) values” in the quantization scheme. Under the hood these quantization schemes still have heavy computational burden in terms of preprocessing: for a very large training data, assigning the weights for each cell will be as difficult as predicting the class label of a query point using \( k \)NN. From this perspective, our denoised bigNN algorithm has the potential to improve quantization methods by shortening the preprocessing time without sacrificing the accuracy (in this paper we have shown the case for the denoising algorithm, a special kind of quantization method).

“Perhaps more discussion/theory on when/why one should use pasting rather than bagging to ensemble \( k \)-NN estimators would be helpful...” Indeed both pasting and bagging can ensemble estimators. Our algorithm is motivated by the need to maintain data decentralisation/privacy and enhance speed performance, while bagging (or bootstrap in general), historically, was proposed to (1) enhance the prediction accuracy by reducing variance and (2) conduct valid statistical inference even when the sample size is not large enough. Neither is our concern here since the sample size is not too small but too large, and by proving the convergence rate we show that the optimal prediction accuracy is retained.

“Separately, depending on the dataset (i.e., the feature space and distribution), I would suspect that even just taking 1 subsample without ensembling could yield a good classifier. Perhaps some discussion on understanding how much training data we could get away with (and whether we could just ignore a lot of the data to save on computation) could be helpful...” Indeed, even Xue and Kpotufe (2018) suggested that a small number of subsamples (repititions) would suffice. Our proof would work even for only one subsample. However, we must point out that denoising and quantization does not work by ignoring a lot of data all together, but by extraditing the information ahead of time (during preprocessing) and not bothering with the entire data later on. As far as the question of how sparse the quantization can be, we had a relevant comment in line 221 which suggested that the smallest each subsample can be is \( N^{1/(2 \alpha + 1)} \).

Reviewer 3 and Reviewer 4: We thank you for your very positive comments.