- 1 We thank the reviewers for their time and observations. The main points raised are:
- 2 1. Our analysis is close to Jin et al.'s, limiting the contribution.
- 3 2. Regularity assumptions pertain to $f \circ R_x$ rather than to f directly.
- 4 3. The algorithm parameters are difficult to determine.

Regarding the first point, we argue that getting the analysis to generalize in a simple way is precisely the contribution. To 5 support the claim that this is not direct, we ask the reviewers to consider the increase in abstraction level and technicality 6 separating Jin et al.'s work from Sun and Fazel's first paper, and also from the more recent work by Sun, Flammarion 7 and Fazel on the same topic (appeared on arXiv after the submission deadline), and to compare this increase to the 8 one separating Jin et al.'s work from our paper. Our more direct generalization, based on a non-standard separation of 9 manifold and tangent steps and on a particular adaptation of the regularity conditions, does not require Riemannian 10 distances, exponentials, logarithms or curvature: all sophisticated objects that require careful considerations (not always 11 spelled out in the existing literature). Yet, our approach yields a more general statement, closer to the original by Jin et 12 al., and, eventually, with less friction. Because of this, there is a better chance that this approach may serve in other 13 contexts as well (we are confident it can extend to stochastic gradients with some work), and that it will be used by 14 others. In our opinion, this is part of the role of a theory-inclined paper: to identify analysis techniques which may be 15 leveraged by others in different contexts, without unnecessary technicalities. 16

About the second point, we note that papers which make Lipschitz-type assumptions about f directly rather than about 17 $f \circ R_x$ typically restrict the algorithm to use a specific retraction (usually, the potentially expensive exponential map 18 which computes geodesics). These Riemannian Lipschitz assumptions typically rely on Riemannian distances, parallel 19 transports along minimizing geodesics, and Riemannian logarithms. The latter are only continuously defined up to the 20 injectivity radius at each point: this radius limitation (which readily appears in the recent work by Sun et al.) should be 21 compared to the role of b here. In contrast, formulating assumptions as we do is straightforward and offers flexibility in 22 choosing the retraction. About assessing whether the regularity assumptions hold, we show that for compact manifolds 23 and smooth f they always do, for any b: this covers a large number of applications. For Euclidean manifolds with 24 $R_x(s) = x + s$, we recover the classical assumptions exactly. In the general case, we show how to handle any retraction 25 by bounding its acceleration, which brings further flexibility. 26

Finally, for the last point, we agree that the need to know the regularity parameters L, ρ, b to run the algorithm is a 27 downside. We explicitly state this early in the paper, and the same issue also affects Jin et al.'s and Sun et al.'s works. 28 However, that certain regularity parameters must be known seems inevitable, in particular for the Hessian's ρ . Indeed, 29 the main theorems make statements about the spectrum of the Hessian, yet the algorithm is not allowed to query the 30 Hessian. If anything, it is remarkable that so little prior knowledge is sufficient. This being said, for structured problems, 31 the parameters can be determined. For example, in PCA, we seek to maximize $f(x) = x^{\top}Ax$ with some symmetric 32 matrix A and x living on the unit sphere $S^{n-1} = \{x \in \mathbb{R}^n : x^\top x = 1\}$. A bit of calculus (which we can add to the appendix) shows that, pullbacked through the classical retraction $R_x(s) = \frac{x+s}{\|x+s\|}$, this cost function satisfies the 33 34 regularity conditions with $L = 2.5 ||A||_{\text{op}}$, $\rho = 9 ||A||_{\text{op}}$ and $b = +\infty$. Of course, $||A||_{\text{op}}$ can be upper-bounded by the subordinate norm $||A||_1$, which is trivial to compute from A. The retraction is second order, hence Assumption 4 holds 35 36 with $\beta = 0$. This is sufficient knowledge to run the algorithm: all other parameters are expressed as functions of these 37

and a user-selected ε .

³⁹ We thoroughly revised the paper and fine details of the statements and proofs to make it a smoother read. In so doing,

40 we further clarified technical points that are already important in the Euclidean case but are not discussed by Jin et al.

⁴¹ We hope the reviewers may re-assess our paper in consideration of these points.