Theory and Applications of Boosting

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Example: "How May I Help You?"

[Gorin et al.]

- goal: automatically categorize type of call requested by phone customer (Collect, CallingCard, PersonToPerson, etc.)
  - `yes I'd like to place a collect call long distance please` (Collect)
  - `operator I need to make a call but I need to bill it to my office` (ThirdNumber)
  - `yes I'd like to place a call on my master card please` (CallingCard)
  - `I just called a number in sioux city and I musta rang the wrong number because I got the wrong party and I would like to have that taken off of my bill` (BillingCredit)

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  - `yes I'd like to place a call on my master card please` (CallingCard)
  - `I just called a number in sioux city and I musta rang the wrong number because I got the wrong party and I would like to have that taken off of my bill` (BillingCredit)
- observation:
  - easy to find "rules of thumb" that are "often" correct
    - e.g.: "IF '`card`' occurs in utterance
      THEN predict 'CallingCard' "
  - hard to find single highly accurate prediction rule

The Boosting Approach

- devise computer program for deriving rough rules of thumb
- apply procedure to subset of examples
- obtain rule of thumb
- apply to 2nd subset of examples
- obtain 2nd rule of thumb
- repeat T times

Details

- how to choose examples on each round?
  - concentrate on “hardest” examples
    (those most often misclassified by previous rules of thumb)
- how to combine rules of thumb into single prediction rule?
  - take (weighted) majority vote of rules of thumb

Boosting

- boosting = general method of converting rough rules of thumb into highly accurate prediction rule
- technically:
  - assume given "weak" learning algorithm that can consistently find classifiers ("rules of thumb") at least slightly better than random, say, accuracy $\geq 55\%$ (in two-class setting)
  - given sufficient data, a boosting algorithm can provably construct single classifier with very high accuracy, say, 99%

Outline of Tutorial

- brief background
- basic algorithm and core theory
- other ways of understanding boosting
- experiments, applications and extensions

Brief Background

Strong and Weak Learnability

- boosting's roots are in "PAC" (Valiant) learning model
- get random examples from unknown, arbitrary distribution
- strong PAC learning algorithm:
  - for any distribution
    with high probability
    given polynomially many examples (and polynomial time)
    can find classifier with arbitrarily small generalization error
- weak PAC learning algorithm
  - same, but generalization error only needs to be slightly better than random guessing ($\frac{1}{2} - \gamma$)
- [Kearns & Valiant '88]:
  - does weak learnability imply strong learnability?

Early Boosting Algorithms

- [Schapire '89]:
  - first provable boosting algorithm
- [Freund '90]:
  - "optimal" algorithm that "boosts by majority"
- [Drucker, Schapire & Simard '92]:
  - first experiments using boosting
  - limited by practical drawbacks

AdaBoost

- [Freund & Schapire '95]:
  - introduced "AdaBoost" algorithm
  - strong practical advantages over previous boosting algorithms
- experiments and applications using AdaBoost:

[Drucker & Cortes '96]
[Jackson & Craven '96]
[Freund & Schapire '96]
[Quinlan '96]
[Breiman '96]
[Maclin & Opitz '97]
[Bauer & Kohavi '97]
[Schwenk & Bengio '98]
[Schapire, Singer & Singhal '98]
[Abney, Schapire & Singer '99]
[Haruno, Shirai & Ooyama '99]
[Cohen & Singer' 99]
[Dietterich '00]
[Schapire & Singer '00]
[Collins '00]
[Escudero, Màrquez & Rigau '00]
[Iyer, Lewis, Schapire et al. '00]
[Onoda, Rätsch & Müller '00]
[Tieu & Viola '00]
[Walker, Rambow & Rogati '01]
[Rochery, Schapire, Rahim & Gupta '01]
[Merler, Furlanello, Larcher & Sboner '01]
[Di Fabbrizio, Dutton, Gupta et al. '02]
[Qu, Adam, Yasui et al. '02]
[Tur, Schapire & Hakkani-Tür '03]
[Viola & Jones '04]
[Middendorf, Kundaje, Wiggins et al. '04]
⋮

- continuing development of theory and algorithms:

[Breiman '98, '99]
[Schapire, Freund, Bartlett & Lee '98]
[Grove & Schuurmans '98]
[Mason, Bartlett & Baxter '98]
[Schapire & Singer '99]
[Cohen & Singer '99]
[Freund & Mason '99]
[Domingo & Watanabe '99]
[Mason, Baxter, Bartlett & Frean '99]
[Duffy & Helmbold '99, '02]
[Freund & Mason '99]
[Ridgeway, Madigan & Richardson '99]
[Kivinen & Warmuth '99]
[Friedman, Hastie & Tibshirani '00]
[Rätsch, Onoda & Müller '00]
[Rätsch, Warmuth, Mika et al. '00]
[Allwein, Schapire & Singer '00]
[Friedman '01]
[Koltchinskii, Panchenko & Lozano '01]
[Collins, Schapire & Singer '02]
[Demiriz, Bennett & Shawe-Taylor '02]
[Lebanon & Lafferty '02]
[Wyner '02]
[Rudin, Daubechies & Schapire '03]
[Jiang '04]
[Lugosi & Vayatis '04]
[Zhang '04]
⋮

Basic Algorithm and Core Theory

- introduction to AdaBoost
- analysis of training error
- analysis of test error based on margins theory

A Formal Description of Boosting

- given training set $(x_1, y_1), \ldots, (x_m, y_m)$
- $y_i \in \{-1, +1\}$ correct label of instance $x_i \in X$

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- for $t = 1, \ldots, T$:
    - construct distribution D_t on $\{1, \ldots, m\}$
    - find weak classifier ("rule of thumb")

      $$h_t : X \to \{-1, +1\}$$

      with small error ϵ_t on D_t:

      $$\epsilon_t = \Pr_{i \sim D_t}[h_t(x_i) \neq y_i]$$

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- output final classifier H_{final}

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[with Freund]

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D_{t+1}(i) &= \frac{D_t(i)}{Z_t} \times \begin{cases} e^{-\alpha_t} & \text{if } y_i = h_t(x_i) \\ e^{\alpha_t} & \text{if } y_i \neq h_t(x_i) \end{cases} \\
&= \frac{D_t(i)}{Z_t} \exp(-\alpha_t \, y_i \, h_t(x_i))
\end{aligned}
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where Z_t = normalization constant

$$\alpha_t = \tfrac{1}{2} \ln\left(\frac{1-\epsilon_t}{\epsilon_t}\right) > 0$$

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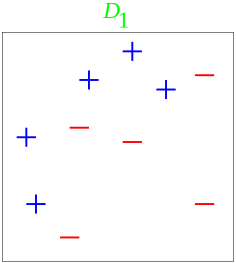
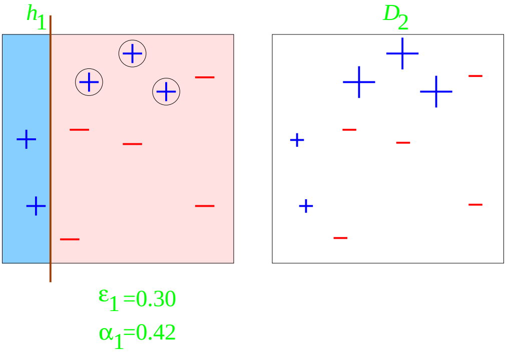
- final classifier:
  - $H_{\text{final}}(x) = \operatorname{sign}\left(\sum_t \alpha_t h_t(x)\right)$

Toy Example

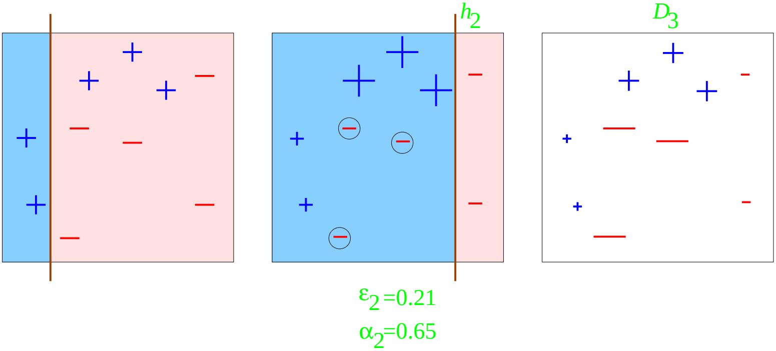


weak classifiers = vertical or horizontal half-planes

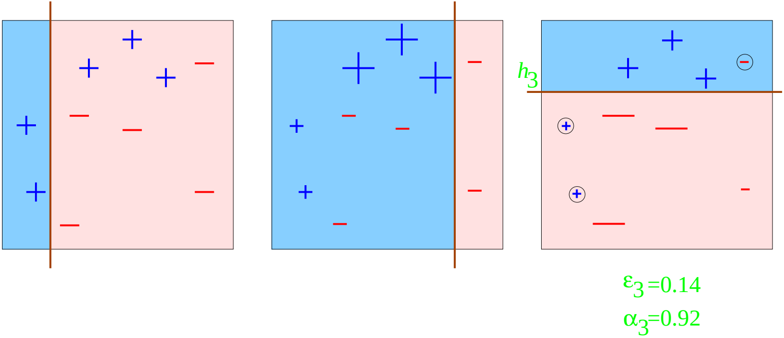
Round 1



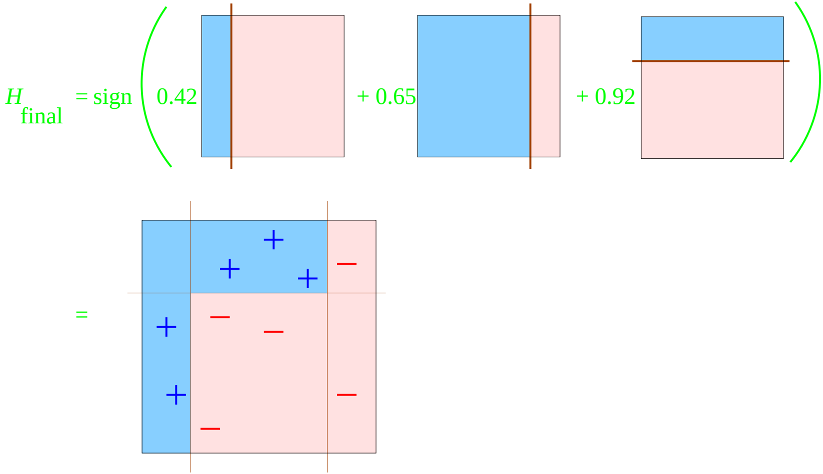
Round 2



Round 3



Final Classifier



Analyzing the training error

- Theorem:
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- so: if $\forall t: \ \gamma_t \geq \gamma > 0$
  then $\text{training error}(H_{\text{final}}) \leq e^{-2\gamma^2 T}$
- AdaBoost is adaptive:
  - does not need to know γ or T a priori
  - can exploit $\gamma_t \gg \gamma$

Proof

- let $f(x) = \sum_t \alpha_t h_t(x) \Rightarrow H_{\text{final}}(x) = \text{sign}(f(x))$
- *Step 1*: unwrapping recurrence:

$$D_{\text{final}}(i) = \frac{1}{m} \frac{\exp\left(-y_i \sum_t \alpha_t h_t(x_i)\right)}{\prod_t Z_t}$$

$$= \frac{1}{m} \frac{\exp\left(-y_i f(x_i)\right)}{\prod_t Z_t}$$

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$$\text{training error}(H_{\text{final}}) = \frac{1}{m} \sum_i \begin{cases} 1 & \text{if } y_i \neq H_{\text{final}}(x_i) \\ 0 & \text{else} \end{cases}$$

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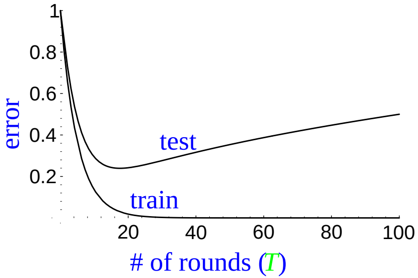
&= \epsilon_t\, e^{\alpha_t} + (1-\epsilon_t)\, e^{-\alpha_t} \\
&= 2\sqrt{\epsilon_t(1-\epsilon_t)}
\end{aligned}$$

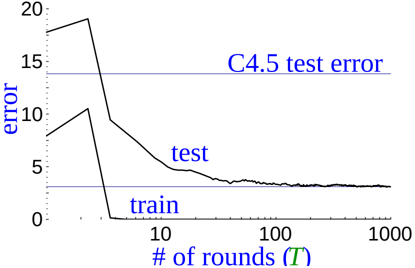
How Will Test Error Behave? (A First Guess)



expect:

- training error to continue to drop (or reach zero)
- test error to increase when H_{final} becomes "too complex"
  - "Occam's razor"
  - overfitting
    - hard to know when to stop training

Actual Typical Run



(boosting C4.5 on "letter" dataset)

- test error does not increase, even after 1000 rounds
  - (total size > 2,000,000 nodes)
- test error continues to drop even after training error is zero!

	# rounds		
	5	100	1000
train error	0.0	0.0	0.0
test error	8.4	3.3	3.1

- Occam's razor wrongly predicts "simpler" rule is better

A Better Story: The Margins Explanation

[with Freund, Bartlett & Lee]

- key idea:
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- key idea:
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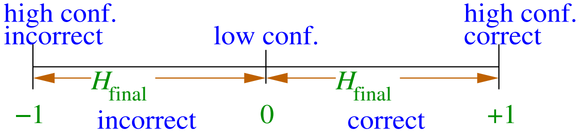
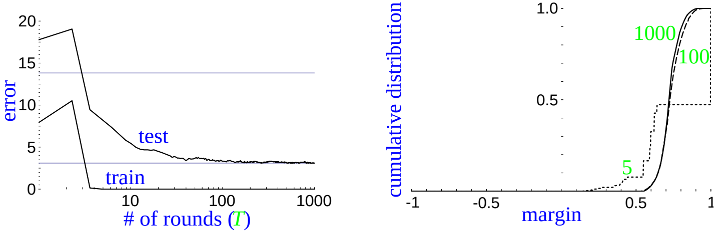
  - should also consider confidence of classifications
- recall: H_{final} is weighted majority vote of weak classifiers
- measure confidence by margin = strength of the vote

  = (fraction voting correctly) − (fraction voting incorrectly)



Empirical Evidence: The Margin Distribution

- margin distribution
  $=$ cumulative distribution of margins of training examples



	# rounds		
	5	100	1000
train error	0.0	0.0	0.0
test error	8.4	3.3	3.1
% margins ≤ 0.5	7.7	0.0	0.0
minimum margin	0.14	0.52	0.55

Theoretical Evidence: Analyzing Boosting Using Margins

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- Theorem: boosting tends to increase margins of training examples (given weak learning assumption)
  - proof idea: similar to training error proof
- so:
  although final classifier is getting larger,
  margins are likely to be increasing,
  so final classifier actually getting close to a simpler classifier,
  driving down the test error

More Technically...

- with high probability, $\forall\theta > 0$:

$$\text{generalization error} \leq \hat{\Pr}[\text{margin} \leq \theta] + \tilde{O}\left(\frac{\sqrt{d/m}}{\theta}\right)$$

($\hat{\Pr}[\,] =$ empirical probability)

  - bound depends on
    - $m = \#$ training examples
    - $d =$ "complexity" of weak classifiers
    - entire distribution of margins of training examples
- $\hat{\Pr}[\text{margin} \leq \theta] \to 0$ exponentially fast (in T) if (error of h_t on D_t) $< 1/2 - \theta$ ($\forall t$)
  - so: if weak learning assumption holds, then all examples will quickly have "large" margins

Other Ways of Understanding AdaBoost

- game theory
- loss minimization
- estimating conditional probabilities

Game Theory

- game defined by matrix $\mathbf{M}$:

	Rock	Paper	Scissors
Rock	$1/2$	1	0
Paper	0	$1/2$	1
Scissors	1	0	$1/2$

- row player chooses row i
- column player chooses column j (simultaneously)
- row player's goal: minimize loss $\mathbf{M}(i, j)$

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- row player's goal: minimize loss $\mathbf{M}(i,j)$
- usually allow randomized play:
  - players choose distributions $\mathbf{P}$ and $\mathbf{Q}$ over rows and columns
- learner's (expected) loss

$$\begin{aligned} &= \sum_{i,j} \mathbf{P}(i)\mathbf{M}(i,j)\mathbf{Q}(j) \\ &= \mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{Q} \equiv \mathbf{M}(\mathbf{P},\mathbf{Q}) \end{aligned}$$

The Minmax Theorem

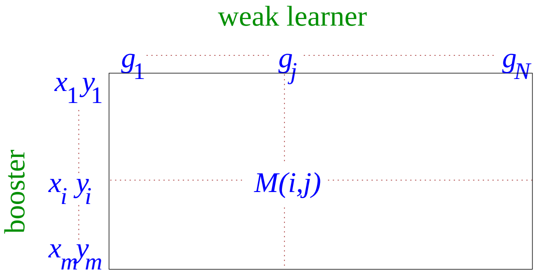
- von Neumann's minmax theorem:

$$\begin{aligned}
\min_{\mathbf{P}} \max_{\mathbf{Q}} \mathbf{M}(\mathbf{P}, \mathbf{Q}) &= \max_{\mathbf{Q}} \min_{\mathbf{P}} \mathbf{M}(\mathbf{P}, \mathbf{Q}) \\
&= v \\
&= \text{“value” of game } \mathbf{M}
\end{aligned}$$

- in words:
  - $v = \min \max$ means:
    - row player has strategy $\mathbf{P}^*$ such that $\forall$ column strategy $\mathbf{Q}$ loss $\mathbf{M}(\mathbf{P}^*, \mathbf{Q}) \leq v$
  - $v = \max \min$ means:
    - this is optimal in sense that column player has strategy $\mathbf{Q}^*$ such that $\forall$ row strategy $\mathbf{P}$ loss $\mathbf{M}(\mathbf{P}, \mathbf{Q}^*) \geq v$

The Boosting Game

- let $\{g_1, \ldots, g_N\}$ = space of all weak classifiers
- row player $\leftrightarrow$ booster
- column player $\leftrightarrow$ weak learner
- matrix **M**:
  - row $\leftrightarrow$ example (x_i, y_i)
  - column $\leftrightarrow$ weak classifier g_j
  - $M(i,j) = \begin{cases} 1 & \text{if } y_i = g_j(x_i) \\ 0 & \text{else} \end{cases}$



Boosting and the Minmax Theorem

- if:
  - $\forall$ distributions over examples
    $\exists h$ with accuracy $\geq \frac{1}{2} + \gamma$
- then:
  - $\min_{\mathbf{P}} \max_{j} \mathbf{M}(\mathbf{P}, j) \geq \frac{1}{2} + \gamma$
- by minmax theorem:
  - $\max_{\mathbf{Q}} \min_{i} \mathbf{M}(i, \mathbf{Q}) \geq \frac{1}{2} + \gamma > \frac{1}{2}$
- which means:
  - $\exists$ weighted majority of classifiers which correctly classifies all examples with positive margin (2γ)
- optimal margin $\leftrightarrow$ "value" of game

AdaBoost and Game Theory

[with Freund]

- AdaBoost is special case of general algorithm for solving games through repeated play
- can show
  - distribution over examples converges to (approximate) minmax strategy for boosting game
  - weights on weak classifiers converge to (approximate) maxmin strategy
- different instantiation of game-playing algorithm gives on-line learning algorithms (such as weighted majority algorithm)

AdaBoost and Exponential Loss

- many (most?) learning algorithms minimize a "loss" function
  - e.g. least squares regression
- training error proof shows AdaBoost actually minimizes

$$\prod_t Z_t = \frac{1}{m} \sum_i \exp(-y_i f(x_i))$$

where $f(x) = \sum_t \alpha_t h_t(x)$

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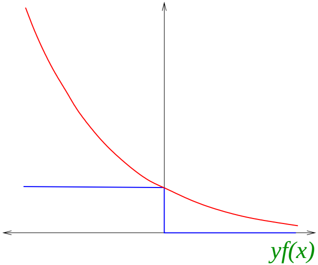
- on each round, AdaBoost greedily chooses α_t and h_t to minimize loss
- exponential loss is an upper bound on 0-1 (classification) loss
- AdaBoost provably minimizes exponential loss



Coordinate Descent

[Breiman]

- $\{g_1, \ldots, g_N\}$ = space of all weak classifiers
- want to find $\lambda_1, \ldots, \lambda_N$ to minimize

$$L(\lambda_1, \ldots, \lambda_N) = \sum_i \exp\left(-y_i \sum_j \lambda_j g_j(x_i)\right)$$

Coordinate Descent

[Breiman]

- $\{g_1, \ldots, g_N\}$ = space of all weak classifiers
- want to find $\lambda_1, \ldots, \lambda_N$ to minimize

$$L(\lambda_1, \ldots, \lambda_N) = \sum_i \exp\left(-y_i \sum_j \lambda_j g_j(x_i)\right)$$

- AdaBoost is actually doing coordinate descent on this optimization problem:
    - initially, all $\lambda_j = 0$
    - each round: choose one coordinate λ_j (corresponding to h_t) and update (increment by α_t)
    - choose update causing biggest decrease in loss
- powerful technique for minimizing over huge space of functions

Functional Gradient Descent

[Friedman][Mason et al.]

- want to minimize

$$L(f) = L(f(x_1), \dots, f(x_m)) = \sum_i \exp(-y_i f(x_i))$$

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[Friedman][Mason et al.]

- want to minimize

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- say have current estimate f and want to improve
- to do gradient descent, would like update

$$f \leftarrow f - \alpha \nabla_f L(f)$$

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$$f \leftarrow f + \alpha h_t$$

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- but update restricted in class of weak classifiers

$$f \leftarrow f + \alpha h_t$$

- so choose h_t "closest" to $-\nabla_f L(f)$
- equivalent to AdaBoost

Benefits of Model Fitting View

- immediate generalization to other loss functions
  - e.g. squared error for regression
  - e.g. logistic regression (by only changing one line of AdaBoost)
- sensible approach for converting output of boosting into conditional probability estimates

Benefits of Model Fitting View

- immediate generalization to other loss functions
  - e.g. squared error for regression
  - e.g. logistic regression (by only changing one line of AdaBoost)
- sensible approach for converting output of boosting into conditional probability estimates
- caveat: wrong to view AdaBoost as just an algorithm for minimizing exponential loss
  - other algorithms for minimizing same loss will (provably) give very poor performance
  - thus, this loss function cannot explain why AdaBoost "works"

Estimating Conditional Probabilities

[Friedman, Hastie & Tibshirani]

- often want to estimate probability that $y = +1$ given x
- AdaBoost minimizes (empirical version of):

$$\mathrm{E}_{x,y}\left[e^{-yf(x)}\right] = \mathrm{E}_x\left[\mathrm{P}\left[y = +1|x\right]e^{-f(x)} + \mathrm{P}\left[y = -1|x\right]e^{f(x)}\right]$$

where x, y random from true distribution

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  where x, y random from true distribution

- over all f, minimized when

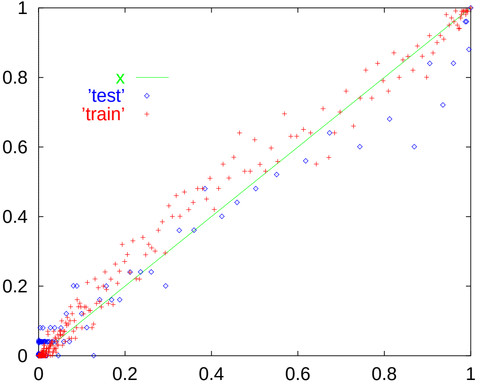
$$f(x) = \frac{1}{2} \cdot \ln\left(\frac{\mathrm{P}\left[y = +1|x\right]}{\mathrm{P}\left[y = -1|x\right]}\right)$$

  or

$$\mathrm{P}\left[y = +1|x\right] = \frac{1}{1 + e^{-2f(x)}}$$

- so, to convert f output by AdaBoost to probability estimate, use same formula

Calibration Curve



- order examples by f value output by AdaBoost
- break into bins of size r
- for each bin, plot a point:
  - x-value: average estimated probability of examples in bin
  - y-value: actual fraction of positive examples in bin

Other Ways to Think about AdaBoost

- dynamical systems
- statistical consistency
- maximum entropy

Experiments, Applications and Extensions

- basic experiments
- multiclass classification
- confidence-rated predictions
- text categorization / spoken-dialogue systems
- incorporating prior knowledge
- active learning
- face detection

Practical Advantages of AdaBoost

- fast
- simple and easy to program
- no parameters to tune (except T)
- flexible — can combine with any learning algorithm
- no prior knowledge needed about weak learner
- provably effective, provided can consistently find rough rules of thumb
  - $\rightarrow$ shift in mind set — goal now is merely to find classifiers barely better than random guessing
- versatile
  - can use with data that is textual, numeric, discrete, etc.
  - has been extended to learning problems well beyond binary classification

Caveats

- performance of AdaBoost depends on data and weak learner
- consistent with theory, AdaBoost can fail if
  - weak classifiers too complex

    $\longrightarrow$ overfitting
  - weak classifiers too weak ($\gamma_t \to 0$ too quickly)

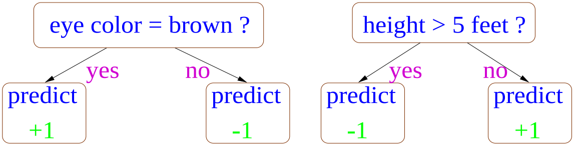
    $\longrightarrow$ underfitting

    $\longrightarrow$ low margins $\longrightarrow$ overfitting
- empirically, AdaBoost seems especially susceptible to uniform noise

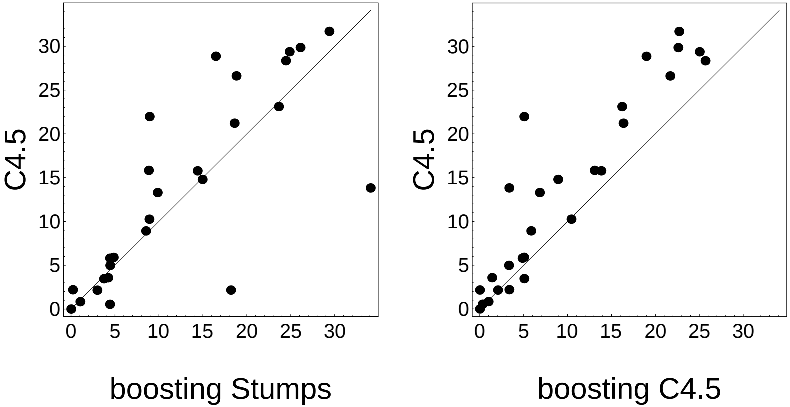
UCI Experiments

[with Freund]

- tested AdaBoost on UCI benchmarks
- used:
  - C4.5 (Quinlan's decision tree algorithm)
  - "decision stumps": very simple rules of thumb that test on single attributes



UCI Results



C4.5
30
25
20
15
10
5
0
0
5
10
15
20
25
30
boosting Stumps
C4.5
30
25
20
15
10
5
0
0
5
10
15
20
25
30
boosting C4.5

Multiclass Problems

[with Freund]

- say $y \in Y = \{1, \ldots, k\}$
- direct approach (AdaBoost.M1):

$$h_t : X \to Y$$

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} \cdot \begin{cases} e^{-\alpha_t} & \text{if } y_i = h_t(x_i) \\ e^{\alpha_t} & \text{if } y_i \neq h_t(x_i) \end{cases}$$

$$H_{\text{final}}(x) = \arg\max_{y \in Y} \sum_{t : h_t(x) = y} \alpha_t$$

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$$H_{\text{final}}(x) = \arg\max_{y \in Y} \sum_{t : h_t(x) = y} \alpha_t$$

- can prove same bound on error if $\forall t : \epsilon_t \leq 1/2$
  - in practice, not usually a problem for "strong" weak learners (e.g., C4.5)
  - significant problem for "weak" weak learners (e.g., decision stumps)
- instead, reduce to binary

Reducing Multiclass to Binary

[with Singer]

- say possible labels are $\{a, b, c, d, e\}$
- each training example replaced by five $\{-1, +1\}$-labeled examples:

$$x \quad , \quad c \quad \rightarrow \begin{cases} (x, a) & , \quad -1 \\ (x, b) & , \quad -1 \\ (x, c) & , \quad +1 \\ (x, d) & , \quad -1 \\ (x, e) & , \quad -1 \end{cases}$$

- predict with label receiving most (weighted) votes

AdaBoost.MH

- can prove:

$$\text{training error}(H_{\text{final}}) \leq \frac{k}{2} \cdot \prod Z_t$$

  - reflects fact that small number of errors in binary predictors can cause overall prediction to be incorrect
- extends immediately to multi-label case (more than one correct label per example)

Using Output Codes

[with Allwein & Singer][Dietterich & Bakiri]

- alternative: choose "code word" for each label

	π_1	π_2	π_3	π_4
a	$-$	$+$	$-$	$+$
b	$-$	$+$	$+$	$-$
c	$+$	$-$	$-$	$+$
d	$+$	$-$	$+$	$+$
e	$-$	$+$	$-$	$-$

- each training example mapped to one example per column

$$x \quad , \quad c \;\rightarrow \begin{cases} (x, \pi_1) & , \quad +1 \\ (x, \pi_2) & , \quad -1 \\ (x, \pi_3) & , \quad -1 \\ (x, \pi_4) & , \quad +1 \end{cases}$$

- to classify new example x:
  - evaluate classifier on $(x, \pi_1), \ldots, (x, \pi_4)$
  - choose label "most consistent" with results

Output Codes (cont.)

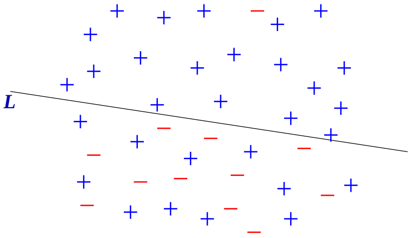
- training error bounds independent of # of classes
- overall prediction robust to large number of errors in binary predictors
- but: binary problems may be harder

Ranking Problems

[with Freund, Iyer & Singer]

- other problems can also be handled by reducing to binary
- e.g.: want to learn to rank objects (say, movies) from examples
- can reduce to multiple binary questions of form:
  "is or is not object A preferred to object B?"
- now apply (binary) AdaBoost

"Hard" Predictions Can Slow Learning



- ideally, want weak classifier that says:

$$h(x) = \begin{cases} +1 & \text{if } x \text{ above } L \\ \text{"don't know"} & \text{else} \end{cases}$$

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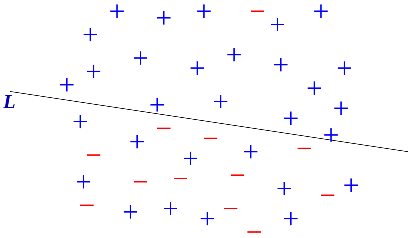

- ideally, want weak classifier that says:

$$h(x) = \begin{cases} +1 & \text{if } x \text{ above } L \\ \text{“don't know”} & \text{else} \end{cases}$$

- problem: cannot express using “hard” predictions
- if must predict ± 1 below L, will introduce many “bad” predictions
    - need to “clean up” on later rounds
- dramatically increases time to convergence

Confidence-rated Predictions

[with Singer]

- useful to allow weak classifiers to assign confidences to predictions
- formally, allow $h_t : X \rightarrow \mathbb{R}$

$$\begin{aligned} \mathrm{sign}(h_t(x)) &= \text{prediction} \\ |h_t(x)| &= \text{"confidence"} \end{aligned}$$

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- formally, allow $h_t : X \to \mathbb{R}$

$$\begin{aligned} \operatorname{sign}(h_t(x)) &= \text{prediction} \\ |h_t(x)| &= \text{“confidence”} \end{aligned}$$

- use identical update:

$$D_{t+1}(i) = \frac{D_t(i)}{Z_t} \cdot \exp(-\alpha_t \; y_i \; h_t(x_i))$$

  and identical rule for combining weak classifiers
- question: how to choose α_t and h_t on each round

Confidence-rated Predictions (cont.)

- saw earlier:

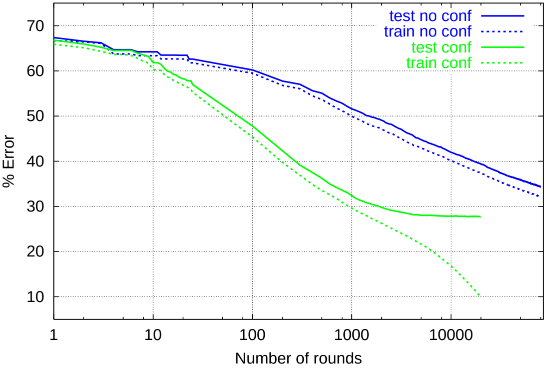
$$\text{training error}(H_{\text{final}}) \le \prod_t Z_t = \frac{1}{m} \sum_i \exp\left(-y_i \sum_t \alpha_t h_t(x_i)\right)$$

- therefore, on each round t, should choose $\alpha_t h_t$ to minimize:

$$Z_t = \sum_i D_t(i) \exp(-\alpha_t \; y_i \; h_t(x_i))$$

- in many cases (e.g., decision stumps), best confidence-rated weak classifier has simple form that can be found efficiently

Confidence-rated Predictions Help a Lot



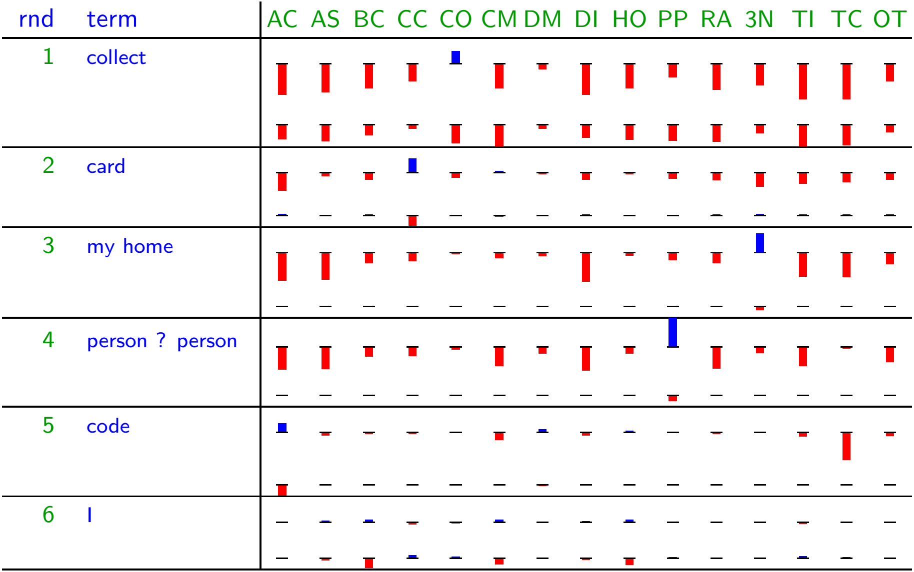
% error	round first reached conf.	round first reached no conf.	speedup
40	268	16,938	63.2
35	598	65,292	109.2
30	1,888	>80,000	–

Application: Boosting for Text Categorization

[with Singer]

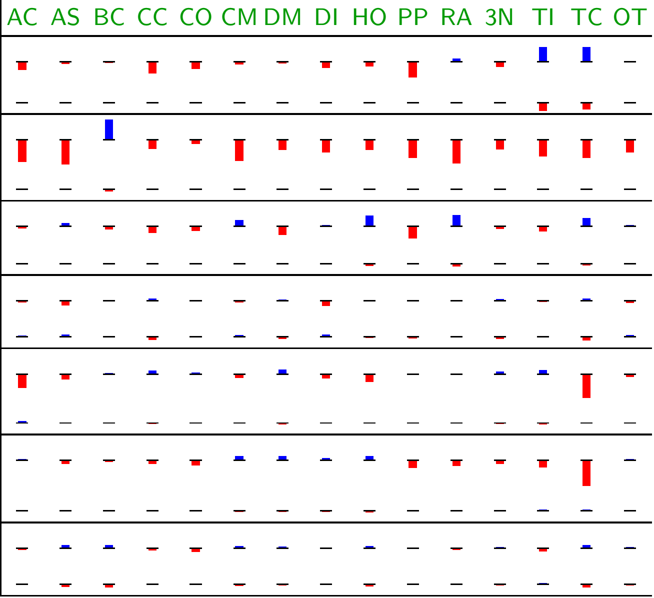
- weak classifiers: very simple weak classifiers that test on simple patterns, namely, (sparse) *n*-grams
  - find parameter α_t and rule h_t of given form which minimize Z_t
  - use efficiently implemented exhaustive search
- "How may I help you" data:
  - 7844 training examples
  - 1000 test examples
  - categories: AreaCode, AttService, BillingCredit, CallingCard, Collect, Competitor, DialForMe, Directory, HowToDial, PersonToPerson, Rate, ThirdNumber, Time, TimeCharge, Other.

Weak Classifiers

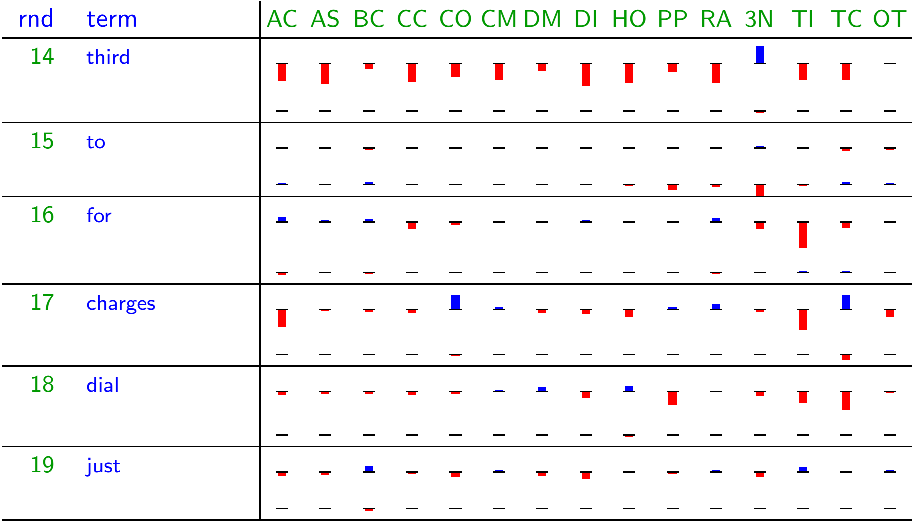


More Weak Classifiers

rnd	term
7	time
8	wrong number
9	how
10	call
11	seven
12	trying to
13	and



More Weak Classifiers



Finding Outliers

examples with most weight are often outliers (mislabeled and/or ambiguous)

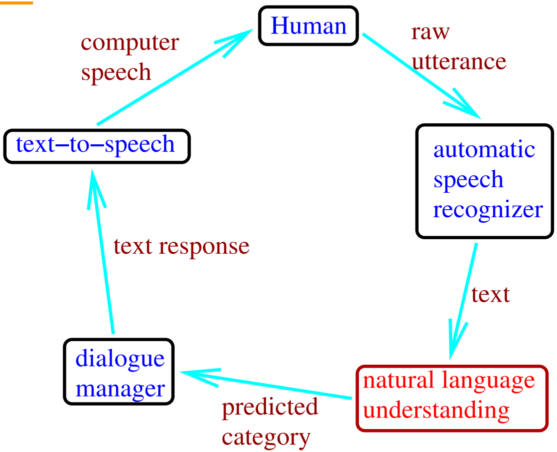
- `I'm trying to make a credit card call` (Collect)
- `hello` (Rate)
- `yes I'd like to make a long distance collect call please` (CallingCard)
- `calling card please` (Collect)
- `yeah I'd like to use my calling card number` (Collect)
- `can I get a collect call` (CallingCard)
- `yes I would like to make a long distant telephone call and have the charges billed to another number` (CallingCard DialForMe)
- `yeah I can not stand it this morning I did oversea call is so bad` (BillingCredit)
- `yeah special offers going on for long distance` (AttService Rate)
- `mister allen please william allen` (PersonToPerson)
- `yes ma'am I I'm trying to make a long distance call to a non dialable point in san miguel philippines` (AttService Other)

Application: Human-computer Spoken Dialogue

[with Rahim, Di Fabbrizio, Dutton, Gupta, Hollister & Riccardi]

- application: automatic “store front” or “help desk” for AT&T Labs’ Natural Voices business
- caller can request demo, pricing information, technical support, sales agent, etc.
- interactive dialogue

How It Works



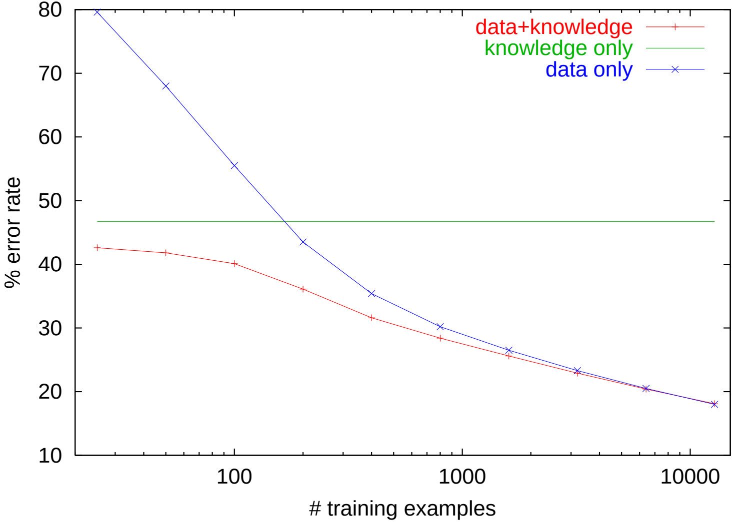
- NLU's job: classify caller utterances into 24 categories (demo, sales rep, pricing info, yes, no, etc.)
- weak classifiers: test for presence of word or phrase

Need for Prior, Human Knowledge

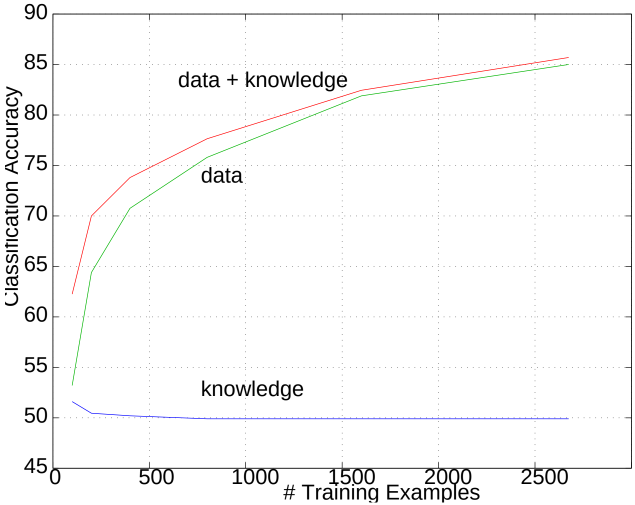
[with Rochery, Rahim & Gupta]

- building NLU: standard text categorization problem
- need lots of data, but for cheap, rapid deployment, can't wait for it
- bootstrapping problem:
  - need labeled data to deploy
  - need to deploy to get labeled data
- idea: use human knowledge to compensate for insufficient data
  - modify loss function to balance fit to data against fit to prior model

Results: AP-Titles



Results: Helpdesk



Problem: Labels are Expensive

- for spoken-dialogue task
  - getting examples is cheap
  - getting labels is expensive
    - must be annotated by humans
- how to reduce number of labels needed?

Active Learning

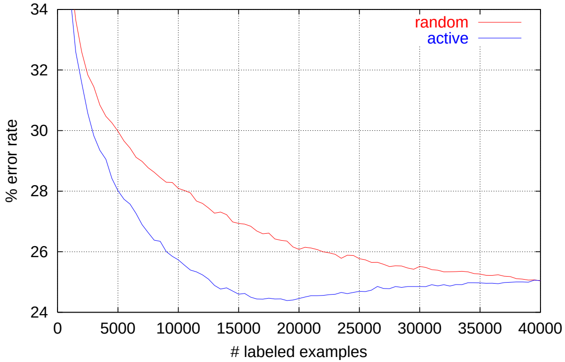
- idea:
  - use selective sampling to choose which examples to label
  - focus on least confident examples [Lewis & Gale]
- for boosting, use (absolute) margin $|f(x)|$ as natural confidence measure

[Abe & Mamitsuka]

Labeling Scheme

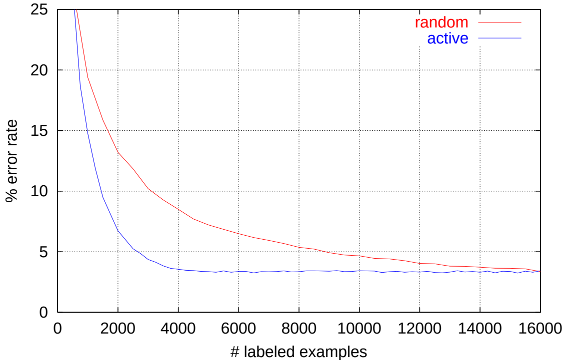
- start with pool of unlabeled examples
- choose (say) 500 examples at random for labeling
- run boosting on all labeled examples
  - get combined classifier f
- pick (say) 250 additional examples from pool for labeling
  - choose examples with minimum $|f(x)|$
- repeat

Results: How-May-I-Help-You?



	first reached		% label
% error	random	active	savings
28	11,000	5,500	50
26	22,000	9,500	57
25	40,000	13,000	68

Results: Letter

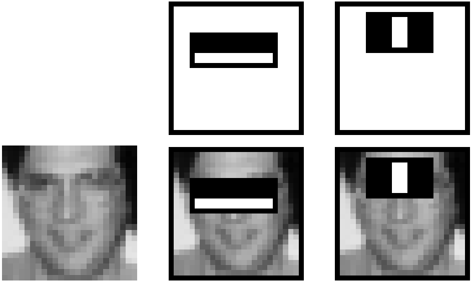


	first reached		% label
% error	random	active	savings
10	3,500	1,500	57
5	9,000	2,750	69
4	13,000	3,500	73

Application: Detecting Faces

[Viola & Jones]

- problem: find faces in photograph or movie
- weak classifiers: detect light/dark rectangles in image



- many clever tricks to make extremely fast and accurate

Conclusions

- boosting is a practical tool for classification and other learning problems
  - grounded in rich theory
  - performs well experimentally
  - often (but not always!) resistant to overfitting
  - many applications and extensions
- many ways to think about boosting
  - none is entirely satisfactory by itself, but each useful in its own way
  - considerable room for further theoretical and experimental work

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